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PROBLEMS IN PHYSICS

FOR

TECHNICAL SCHOOLS, COLLEGES,
AND UNIVERSITIES

BY

WILLIAM D. HENDERSON, PH. D.,
ASSOCIATE PROFESSOR OF PHYSICS, UNIVERSITY OF MICHIGAN

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PREFACE

This text has grown out of some seven years' experience in the teaching of large classes of engineering students in general physics in the University of Michigan. Throughout the course of this work *one hour a week was devoted wholly to the solution of practical problems bearing upon the fundamental principles treated in the lecture room and laboratory.* This phase of the work proved so satisfactory that it was deemed desirable to incorporate the typewritten exercises into the form of the present volume. This was done both for the convenience of our own students and instructors, and also with the hope that the exercises may be of service to other teachers of physics. These exercises are intended to supplement the usual one year's course in general physics, with a supply of material of such range and variety as will be likely to stimulate the student's interest and clarify his understanding.

The chief characteristics of this text may be summarized briefly as follows:

1. *Statement of Fundamental Principles.*—Accompanying each set of problems there is a brief statement of the fundamental principles involved, and also a large number of illustrative examples which enable the student to proceed with his work with a minimum of time and attention on the part of the instructor.

2. *Character of the Problems.*—The problems are practical, carefully graded, and are thoroughly workable.

3. *Range of Problems.*—The one thousand and twenty-five problems herein contained offer a range and variety which will enable the instructor not only to select examples suitable for special groups of students, but also to vary the assignments from year to year.

4. *Data Modern.*—The data presented in connection with these exercises are thoroughly modern and in accordance with the recommendations and practice of the United States Bureau of Standards, and of our Scientific Societies and Engineering Associations.

5. *Problems Original.*—Most of the problems of this set are original, having been written by the author, in conjunction with

Prof. N. H. Williams of the Department of Physics, University of Michigan. It should be stated, however, that in the preparation of the exercises many sources of information have been drawn upon, acknowledgment in each individual case not always being considered necessary, or convenient.

The author cannot hope that these problems are entirely free from errors. He will consider it a favor, therefore, if teachers who use the text will inform him of corrections or criticisms.

The author wishes to thank Professors N. H. Williams and D. L. Rich, both of the Department of Physics, University of Michigan, for helpful criticisms and valuable suggestions.

W. D. H.

ANN ARBOR, MICHIGAN.
August, 1916.

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PROBLEMS IN PHYSICS

CHAPTER I

MECHANICS OF SOLIDS

UNITS OF MEASUREMENT

1. Fundamental Units.—The fundamental units of measurement are those of length, mass, and time. In the United States the legal units of length and mass are derived from the standards of the metric system, which was legalized by Act of Congress in 1866. The English units of this country (the foot, the pound, etc.) come historically from the units of Great Britain; *they are, however, defined in terms of the metric system.* The U. S. English units of length and mass, therefore, differ somewhat from the corresponding British units. For example, the U. S. inch is 1/39.37 of a standard meter; the British inch, 1/36 of a standard British yard. The U. S. inch is just a little longer than the British inch, as may be seen from the following: 1 meter = 39.37 U. S. inches = 39.37079 British inches.

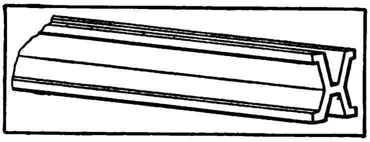


FIG. 1.—Section of U. S. standard meter.

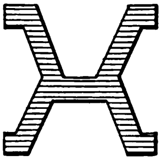


FIG. 2.—Cross-section of U. S. standard meter, actual size.

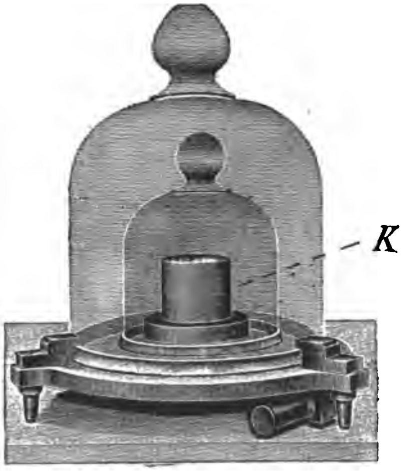


FIG. 3.—U S. standard kilogram.

2. Metric Standards.—The International metric standards of length and mass are kept at the International Bureau, at Sèvres, near Paris, France. The U. S. metric standards are the National meter and kilogram, kept at the Bureau of Standards, Washington, D. C. Our national metric standards are, as nearly as possible, exact copies of the international standards. There is shown in Fig. 1 a reproduction of a photograph of the U. S. national meter;

in Fig. 2, a cross-section of the national meter (actual size); in Fig. 3, the U. S. national kilogram.

3. Spelling and Abbreviation of Units.—According to Circular No. 47, issued July 1, 1914, the National Bureau of Standards offers certain recommendations with reference to the spelling and abbreviations of units. Among the rules adopted by the Bureau are the following:

1. "The period is omitted after abbreviations of metric units, while it is used after those of the customary system." For example, we should write *cm* for centimeter, *mm* for millimeter, *m* for meter, and so on; in the case of units of the English system, however, we write *ft.* for foot, *lb.* for pound, *cu. ft.* for cubic foot, etc.

2. "The exponents ^{2} and ^{3} are used to signify area and volume respectively, in the case of the metric units, instead of the longer prefixes sq. cm. or cu. cm." In accordance with this rule, therefore, we should write cm^2 for square centimeter, and cm^3 for cubic centimeter. It must be noted in this connection, however, that it is customary in chemical practice, as well as in physics, to employ the symbol *cc* to designate the milliliter; that is, 1/1000 of a liter.

3. "The use of the same abbreviations for both singular and plural is recommended." Thus, *cm* may stand for centimeter or centimeters.

4. Metric Units of Length.—The metric unit of length is the meter. A meter is the distance between two points on a platinum iridium bar kept at the Bureau of Standards at Washington, the measurement being made at 0°C. In designating fractions of the meter we use the Latin prefixes, *deci*, *centi*, *milli;* multiples, the Greek prefixes, *deka*, *hecto*, *kilo*. The divisions and multiples of the meter are given in the following tables:

Fractions			Multiples		
1 decimeter (dm)	= 1/10	meter	1 dekameter (dkm)	= 10	meters
1 centimeter (cm)	= 1/100	meter	1 hectometer (hm)	= 100	meters
1 millimeter (mm)	= 1/1000	meter	1 kilometer (km)	= 1,000	meters

5. Metric Unit of Volume.—The metric unit of volume is the cubic meter (m^3). The fractional units of volume are the cubic decimeter (dm^3) and the cubic centimeter (cm^3).

6. Metric Unit of Capacity.—The metric unit of capacity is the liter. A liter is the volume of 1 kg of air-free distilled water at 4°C. The reason for defining the liter in terms of the volume of a kilogram of air-free distilled water at 4°C, instead of defining it directly as 1,000 cc is because of the convenience in calibrating glass flasks and similar vessels.

$$1 \text{ liter} = 1{,}000 \text{ cc} = 1{,}000.027 \text{ cm}^3$$
$$1 \text{ cc} = 1/1000 \text{ liter} = 1 \text{ milliliter (ml)}.$$

Since a kilogram of water at 4°C has a volume which is very nearly equal to a cubic decimeter, a liter will be considered as equivalent to 1,000 cm^3 unless specifically stated to the contrary.

Bureau of Standards Circular No. 9 contains the following statement:

"In all volumetric work the unit of volume employed is the *milliliter* and not the cubic centimeter, but in this country it is ordinarily designated by the letters *cc*. This designation is so firmly established both in the trade and in laboratory practice that it is for the present retained in this circular, though in strictness the abbreviation *ml* should be used. It is hoped that the correct usage may soon replace the present inexact use of the term *cc.*"

7. Metric Units of Mass.—The U. S. standard of mass is the kilogram. A kilogram is a mass equivalent to the National Standard Kilogram. A gram is 1/1000 of a kilogram. For all practical purposes a gram may be considered as equivalent to the mass of a cubic centimeter of air-free distilled water. The divisions of the kilogram and gram are given in the following tables:

1 gram (g) = 1/1000 kilogram (kg)
1 decigram (dg) = 1/10 gram
1 centigram (cg) = 1/100 gram
1 milligram (mg) = 1/1000 gram

8. Unit of Time.—The unit of time is the second. A second is 1/86,400 of a mean solar day. A solar day is the interval between two successive passages of the sun across a given meridian. Solar days vary in length throughout the year. A mean solar day is the average length of all the solar days taken throughout the year. The time recorded by watches and clocks is expressed in mean solar time.

9. Metric Units and U. S. English Equivalents.—The following table contains the metric units of length, capacity, and mass and their legal English equivalents.

Metric Units		English Equivalents
1 meter	=	39.37 inches
1 liter	=	1.0567 quarts (liquid measure)
1 liter	=	0.908 quarts (dry measure)
1 kilogram	=	2.204622 pounds
1 gram	=	15.432 grains

10. U. S. English Units and Metric Equivalents.—The following table contains the U. S. English units of length, Fig. 4, capacity, and mass, together with certain recently adopted standards.

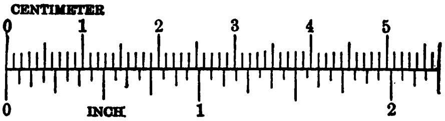


FIG. 4.—Relation of inch to centimeter.

U. S. English Units
1 inch = 1/39.37 meter = 2.54 centimeters
1 quart (L.M.) = 946.333 cubic centimeters
1 quart (D.M.) = 1,101.2 cubic centimeters
1 pound = 1/2.204622 kilogram = 453.592 grams
1 gallon = 231 cubic inches
1 bushel = 2,150.42 cubic inches
1 standard barrel = 7,056 cubic inches

11. U. S. Coins.—According to the rules of the United States Mint, the following pieces of money, as legalized by act of Congress, are coined in terms of the metric units of mass. The mass of a

5-cent piece (nickel) = 5.0 grams,
10-cent piece (dime) = 2.5 grams,
25-cent piece (quarter) = 6.25 grams,
50-cent piece (half dollar) = 12.5 grams.

Problems

NOTE.—In the solution of all problems involving the ratio 3.1416 the symbol π should be used, unless stated to the contrary.

Example.—Find the area of a circle the radius of which is 10 in.

Ans. *Area* = 100π *sq. in.*

1. (*a*) Find the value of 2.6 km in meters, centimeters, millimeters. (*b*) Find the value of 104 cm in millimeters, meters, kilometers.

2. In 10 miles there are how many rods? yards? feet? kilometers?

3. The ordinary country highway is 4 rd. wide. This is equivalent to how many feet? meters?

4. At the 1912 Olympiad held in Stockholm, Sweden, the "100-meter dash" was won in the record time of 10.6 sec. At this rate, find the average speed of the runner in (*a*) feet per second; (*b*) miles per hour.

5. The great cannon used by the Germans in the reduction of the Belgium forts in 1914 were known as "42-centimeter guns." Find the diameter of the bore of these guns in inches.

6. The train known as the "Twentieth Century Limited" of the New York Central R. R. is scheduled to run from Chicago to New York, a distance of 1,003 miles in 20 hr. Find the average speed of this train in (*a*) miles per hour; (*b*) kilometers per hour; (*c*) feet per second; (*d*) meters per second.

7. The best speed ever made by a vehicle running on rails, up to the present time (1916), was that recorded in the Berlin-Zossen tests of electric cars, Oct. 27, 1903, when a rate of 210 km per hr. was attained. Find the speed attained by this car in miles per hour.

8. The highest speed (1916) ever travelled on the surface of the earth in a vehicle was made in an automobile run during a speed test over the hard and level surface of the crystallized salt beds at Salduro, Utah, Aug. 12, 1915. The best time for 1 mile was 25.2 sec. Find the equivalent speed in (*a*) miles per hour; (*b*) kilometers per hour.

9. The radius of a circle is 10 cm. Find the area of this circle in (*a*) square centimeters; (*b*) square inches.

10. Find the cross-sectional area of the bore of a 42-cm gun in (*a*) square centimeters; (*b*) square inches.

11. According to Act of Congress, 1915, the diameter of the "head" of a standard barrel for "fruits, vegetables, and other dry commodities," shall be 17⅛ in. Find the area of the end of this barrel in (*a*) square inches; (*b*) square centimeters.

12. The circumference of a cylindrical water tank is 20π ft. Find in feet and meters (*a*) the radius; (*b*) the cross-sectional area.

13. The height of the tank (problem 12) is 20 ft. Find in square feet (*a*) the lateral area of the tank; (*b*) the total area.

14. Find the volume of the tank (problem 12) in (*a*) cubic feet; (*b*) liters.

15. Find the capacity of the tank (problem 12) in gallons (liquid measure).

16. A cubic foot of water weighs 62.4 lb. Find the weight of the water in the tank (problem 12) when it is full, (*a*) in pounds; (*b*) in kilograms.

17. The radius of a sphere is 10 cm. Find (*a*) its area in square centimeters; (*b*) its volume in cubic centimeters.

18. The radius of a sphere is 1 ft. 8 in. Find (*a*) its area in square inches; (*b*) its volume in cubic inches.

19. Find the capacity of the sphere (problem 18) in (*a*) quarts; (*b*) liters.

20. The volume of a sphere is 2304π cu. in. Find in square feet (*a*) the area of a sphere; (*b*) the area of a great circle of the sphere.

21. The radius of a base of a right cone is 4 ft. Its height is 10 ft. Find (*a*) the lateral area of the cone; (*b*) the total area; (*c*) the volume.

22. The cone (problem 21) is filled with water. Find the weight of the water in (*a*) pounds; (*b*) kilograms.

23. The spherical bulb of a thermometer is 0.6 cm in diameter (inside measurement). It is filled with mercury, density 13.59 grams per cm^3. Find in grams the mass of the mercury in the bulb.

24. Twelve bicycle balls, each having a diameter of 1 cm are dropped into a liquid having a density of 0.8 grams per cm^3. Find (*a*) the volume of the liquid displaced; (*b*) the mass in grams of the liquid displaced.

25. An overflow vessel is full of distilled water having a temperature of 4°C. Into this vessel there is dropped six spherical metal balls of uniform size, causing a displacement of 13.824π grams of water. Find the diameter of one of the balls.

26. Out of a circular piece of metal of radius 10 in. there is cut a sector having an arc of 1 ft. Find the area of the sector.

27. According to the rules of the U. S. Mint, a 5-cent nickel coin has a mass of 5 grams; a 10-cent piece, a mass of 2.5 grams; a 25-cent piece, a mass of 6.25 grams. Find the weight in pounds of $10 in (*a*) nickels; (*b*) dimes; (*c*) quarters.

SOME FUNDAMENTAL EQUATIONS

12. Dimensional Formulæ.—It is frequently of advantage to express physical quantities in terms of length (L), mass (M), and time (T). For example, the dimensional formula for an area (a length multiplied by a length) is L^2; likewise the dimensional formula for a volume is L^3. Density is equal to mass per unit volume; that is, $D = M/V$. The dimensional formula for density, then, is $M/L^3 = ML^{-3}$.

13. Angular Measure.—An angle may be measured in degrees or radians. In a complete circle there are 360°, or 2π radians. Suppose that the radius r, Fig. 5, rotates about the point O from A to A', sweeping out the angle θ. The angle θ is measured by the arc s and may be expressed in radians or degrees. In this text, unless stated to the contrary, angles designated by the Greek letters θ, ϕ, α, ω, etc., represent angles measured in *radians*. In a complete circle there are 2π radians, or 360°.

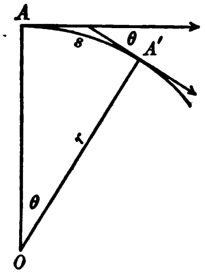


FIG. 5.—Curvature.

$$\text{radians} = \text{arc}/\text{radius} = s/r$$
$$2\pi \text{ radians} = 360°$$
$$1 \text{ radian} = 180/\pi = 180/3.1416 = 57.296°$$

14. Curvature.—Curvature is the space rate of change of direction. From Fig. 5 we may write,

$$\textit{Curvature} = \textit{angle}/\textit{arc} = \theta/s = 1/r$$

15. The Right-angled Triangle.—Trigonometrical functions relate primarily to the relation of the sides of a right-angled triangle to the direction angle (θ), Fig. 6.

$$\sin\theta = c/b;\ \cos\theta = a/b;\ \tan\theta = c/a$$

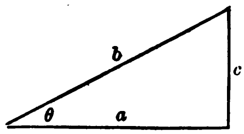


FIG. 6.—Functions of the angle θ.

16. Projection on Rectangular Axes.—It is frequently of importance to get the projection of a given vector quantity upon the x- and y-axes.

For example, the projections of b upon the x- and y-axes are represented by the quantities a and c', Fig. 7; that is

$$a = b \cos \theta, \text{ and } c' = b \sin \theta$$

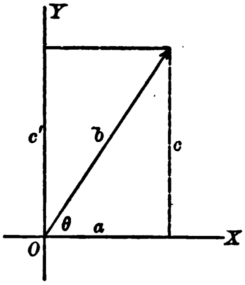


Fig. 7.—Projection on X- and Y-axis.

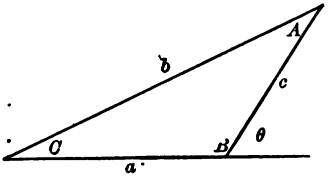


Fig. 8.—Relation between sides and angles of a triangle.

17. Sides and Angles of a Triangle.—In solutions involving the sides and angles of a triangle, Fig. 8, there are three important equations which should be learned by the students. These are:

$$b^2 = a^2 + c^2 + 2ac \cos \theta,$$
$$b^2 = a^2 + c^2 - 2ac \cos B,$$
$$a/\sin A = b/\sin B = c/\sin C$$

Example.—Consider the triangle, Fig. 8. The side a is 20 units; c is 15 units; and the θ is 60°. Find the magnitude and direction of b, with respect to a.

Solution.—$b^2 = 400 + 225 + 2 \times 300 \times 0.5$. *Hence the magnitude of* $b = 30.4$. *Now the direction of* b *is determined by the angle* C. *Angle* $B = 120°$; *then* $30.4/0.866 = 15/\sin C$. *Angle* $C = 25° 18'$.

The composition and resolution of velocities, accelerations, and forces, or any other factors which may be treated by the vectorial method, may be considered as problems in trigonometry, in which we have to deal with the sides and angles of a triangle.

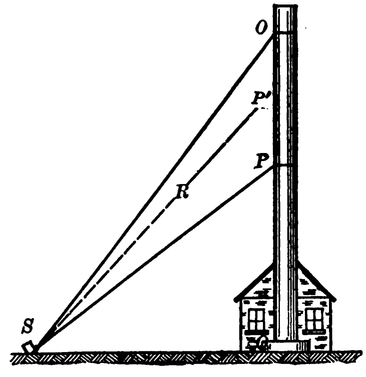


Fig. 9.—Components of wind pressure on a smokestack.

Example.—Suppose that a smokestack, Fig. 9, is supported in a given plane by two steel cables which are fastened to a staple S, attached to a post. At a given time the wind pressure is so distributed that the force exerted on SO is 240 lb., and the force exerted on SP is 180 lb. The angle OSP is 20°. We wish to find the magnitude and direction of the resultant force acting on the staple S. We represent the two forces (180 and 240) as the sides of a triangle, a and c, Fig. 10, the angle θ being 20°. Our problem is to find the side b, and the angle C.

Solution.—By equation $b^2 = a^2 + c^2 + 2ac \cos \theta$, $b^2 = 180^2 + 240^2 + 2 \times 180 \times 240 \times 0.9397$. *Hence* $b = 413.7$, *and by equation of Art.* 17, *angle* $C = 10° \, 41'$.

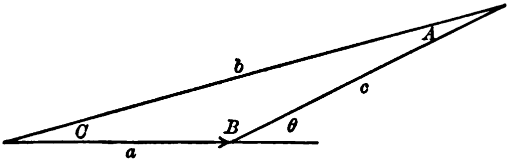


FIG. 10.

Problems

28. Velocity (v) = length/time. Show that the dimensional formula for velocity is LT^{-1}.

29. Acceleration (a) = velocity/time. Show that the dimensional formula for acceleration is LT^{-2}.

30. Force (F) = mass × acceleration. Show that the dimensional formula for force is MLT^{-2}.

31. Momentum (mv) = mass × velocity. Write the dimensional formula for momentum.

32. An angle (θ), measured in radians, = arc/radius. Explain what is meant when we say that the dimensional formula for θ is $L_yL_x^{-1}$.

33. Show that the dimensional formula for curvature (θ/s) = L^{-1}.

34. A body moving in a circle passes over an arc of 20 cm in a given time. Find the curvature when the angle swept out is (a) 40°; (b) $\pi/8$ radians. Be sure to name the units in which your answers are expressed.

35. Find the curvature of a body moving in a circle when the radius is (a) 12 cm; (b) 12 in.; (c) 1 ft.

36. A body moves in a circle, through an arc of 2 ft., sweeping out an angle of 30°. Find the curvature.

37. Two forces, F and F', act on a body at the point O, F to the eastward (right) and F' to the northeast, making an angle of 45° with F. The magnitude of F is 30 lb.; that of F', 20 lb. Find the magnitude and direction of the resultant R. Make sketch to illustrate the solution of the problem.

38. Find the vertical and horizontal components of the resultant R (problem 37).

39. Consider the resultant force *R* acting on the staple *S*, Fig. 9, to be 500 lb. The distance *QP′* is 80 ft.; the distance *SQ* is 110 ft. Find (*a*) the angle which the resultant *R* makes with the ground *SQ*; (*b*) the horizontal component of the force acting on the staple *S*; (*c*) the vertical component.

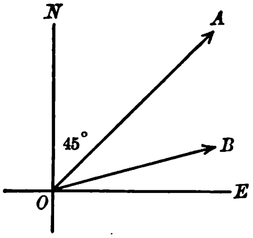


FIG. 11.

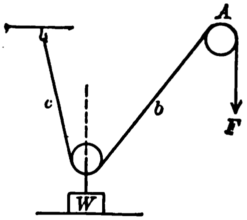


FIG. 12.

40. A man rows a boat in a northeasterly direction *OA*, Fig. 11, with a velocity of 3 miles per hr. At the same time the boat drifts, due to the wind, in the direction *OB* with a velocity of $2\frac{14}{15}$ ft. per sec. The angle *AOB* is 30°. Find (*a*) the magnitude of the resultant velocity and (*b*) the direction in which the boat actually moves with reference to an east-west line.

41. Find (problem 40) (*a*) the easterly, and (*b*) the northerly velocity of the boat.

42. Suppose that the man (problem 40) rows the boat in the direction *OA* with a velocity of 3 miles per hr., and the wind and currents carry him westward at the rate of 3 miles per hr. How far west of *O*, Fig. 11, will he be in 1 hr.?

43. Suppose that a force *F*, of 100 lb., Fig. 12, acts vertically on the rope passing over the pulley *A*. The rope *b* makes an angle with the vertical at *W* of 30°, and *c* makes an angle of 10°. Neglecting the friction of the pulleys, and assuming that the force applied at *F* is transmitted undiminished to all parts of the rope, find (*a*) the force with which *W* is lifted vertically; (*b*) the magnitude of the horizontal component of the force acting on *W*.

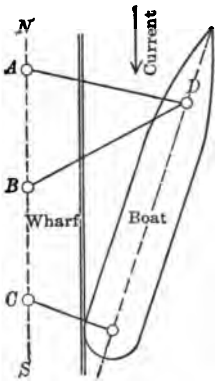


FIG. 13.—Components of water pressure.

44. A boat is moored to a wharf as shown in Fig. 13. The current is running "bow on," so that the bow of the boat tends to swing outward from the wharf. The force acting on the cable *DA* is 200 lb., and that on *DB* is 300 lb. The angle *DAB* is 80°; *DBC* is 100°. Find (*a*) the magnitude and (*b*) the direction of the resultant of the forces acting at *D*.

45. Find (*a*) the west, and (*b*) the south components of the resultant force acting on *D* of problem 44.

46. Suppose that a body at *O* has impressed upon it a force *OA* of 120 lb., and a force *OB* of 80 lb., the two making an angle of 120°. Force *OA* makes an angle of 30° with *OX*. Find (*a*) the magnitude and (*b*) the direction of the resultant *R*, with reference to *OX*.

47. The resultant of two forces *OA* and *OB* has a magnitude of 600 lb., and makes an angle of 30° with *OA*, the value of which is 300 lb. Find the magnitude of *OB*.

48. A hill used for coasting in winter has a rise of 1 ft. in 4; that is, a vertical rise of 1 ft. for a horizontal distance of 4 ft. A boy sliding down the hill has at a given instant a velocity of 20 miles per hr. Find (*a*) his vertical velocity, and (*b*) his horizontal velocity.

ACCELERATION

18. Definition and Formulæ.—Acceleration is the change (increase or decrease) of velocity per unit of time. When the acceleration is positive we use the symbol $+a$; when negative, $-a$. Let v' be the initial which a body has at a given instant; v, the final velocity at the end of the time t; and a, the acceleration. Let s be the space passed over in the time t. We may write the fundamental equations connecting v, a, s, and t, as follows:

$$v = v' \pm at$$
$$s = v't \pm \tfrac{1}{2}at^2$$
$$v^2 = v'^2 \pm 2as$$

In the case of falling bodies, the acceleration is represented by the symbol g. The numerical value of the acceleration due to gravity, employed in the solution of problems in this text, is $g = 32$ ft. per sec. per sec. $= 980$ cm per sec. per sec.

The resultant of two or more accelerations is the vectorial sum of the components. For example, if an elevator move downward with an acceleration of 8 ft. per sec. per sec., the resultant or total acceleration is $32 + 8 = 40$ ft. per sec. per sec. If the elevator start to move upward with a negative acceleration of 8 ft. per sec. per sec., the resultant acceleration is $32 - 8 = 24$ ft. per sec. per sec.

Problems

49. A body starts from rest and slides down an inclined plane with uniform accelerated motion. At the end of the first second its velocity is 10 cm per sec.; at the end of the second second, 20 cm per sec.; and the third, 30 cm per sec., and so on. (*a*) What is the acceleration? Is it positive or negative? (*b*) What is the velocity at the end of 10 sec.? (*c*) The space passed over in 10 sec.?

50. Consider a body as moving down an inclined plane for 10 sec., with an acceleration of 10 cm per sec. per sec. (*a*) What is its velocity at the end of the fifth second? At the end of the tenth second? What is the initial velocity at the beginning of the sixth second?

(*b*) What is the average velocity during the first 5 sec.? During the last 5 sec. (sixth to tenth seconds inclusive)? (*c*) How far does the body move during the first 5 sec.? During the last 5 sec.?

51. A body falls from rest under the force of gravity for 10 sec. Find the final velocity, and the space passed over in (*a*) centimeters; (*b*) feet.

52. A body is thrown vertically downward with an initial velocity of 10 ft. per sec. How far will it fall in (*a*) 5 sec.; (*b*) during the fifth second?

53. A body is projected vertically upward with an initial velocity of 320 ft. per sec. (*a*) In what time will it come to rest? (*b*) How high will it rise?

54. A body is projected vertically upward with an initial velocity of 8,820 cm per sec. How high above the starting point will it be in (*a*) 6 sec.? (*b*) 9 sec.? (*c*) 12 sec.?

55. A body is projected vertically upward with an initial velocity of 6,860 cm. per sec. Find in what time it will be 22,050 cm above the starting point. How do you account for the two values of t?

56. A body starting from rest, is acted upon by a constant force. At the end of the first second its velocity is 5 ft. per sec. (*a*) Find velocity at end of tenth second. (*b*) How far did it travel during the 10 seconds? (*c*) During the tenth second?

57. (*a*) What is the velocity of the body (problem 56) at the beginning of the seventh second? (*b*) How far did it travel during the seventh second? (*c*) If all force had ceased to act on it at the end of the tenth second, how far would it have travelled the eleventh second?

58. A ball is projected upward with an initial velocity of 160 ft. per sec. Consider the acceleration of gravity (g) to be 32 ft. per sec. per sec. (*a*) How high will the ball rise? (*b*) Velocity at end of 4 sec.? (*c*) At end of 6 sec.?

59. (*a*) How far will the ball (problem 58) travel the first second? (*b*) the fifth second? (*c*) the seventh second?

60. A train running at the rate of 36 miles per hour comes to rest in 10 seconds. (*a*) Find acceleration in feet per second per second. (*b*) What distance did it travel during the 10 seconds?

61. A train 300 ft. long passes another train of equal length, having an equal velocity, and going in an opposite sense. The two trains are clear of each other in 10 sec. (*a*) What is the velocity of the train with respect to the track? (*b*) With respect to the other train?

62. A body is projected upward with a velocity of 96 ft. per sec. Where will it be at the end of (*a*) 2 sec.? (*b*) 4 sec.? (*c*) When will it reach the ground?

63. A sled starting from rest runs down hill with a uniformly accelerated motion. Its velocity at the end of the fourth second is 20 ft. per sec. Find (*a*) the acceleration; (*b*) the velocity at the end of the tenth second; (*c*) the space passed over during the 10 seconds; (*d*) space passed over during the tenth second.

64. A stone is thrown downward with an initial velocity of 10 ft. per sec. Find (*a*) its velocity at the end of 10 seconds; (*b*) the distance traversed during the 10 seconds; (*c*) the distance during the tenth second.

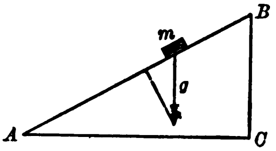


Fig. 14.—Motion on inclined plane.

65. A train running at the rate of 30 miles per hr. comes to rest in 8 sec. (*a*) Find the acceleration in feet per second per second. (*b*) What distance did it travel during the 8 seconds?

66. A bullet is shot horizontally from a lighthouse tower 64 ft. high, with a velocity of 400 ft. per sec. Where will it strike the water? (Neglect air resistance.)

67. A body slides without friction down an incline, as shown in Fig. 14. Consider the mass *m* of the body to be 10 grams and the value of *g* 980; angle *ABC* 60°. Find (*a*) the acceleration down the incline; (*b*) the velocity of the end of the fourth second; (*b*) the distance passed over during the fourth second.

FORCE

19. Units of Force.—Force is that which produces or tends to produce motion. Force may be measured in two ways: First, by the push or pull which it exerts; second, by the acceleration which it imparts to a given mass. The pull which a force exerts may be measured by means of a spring balance (dynamometer); in the second case the magnitude of the force may be measured in terms of the product of the mass times the acceleration, as represented by the equation, $F = ma$.

Pressure is Force per unit area; that is $P = F/A$.

There are two sets of units of force, known as (*a*) gravitational or practical units, and (*b*) absolute units. Practical and absolute units may be expressed in both the English and metric system. The relation of the two sets of units is shown in the following outline:

Units of Force
- *Gravitational*
  - *English* = *pound of force = weight of a pound.*
  - *Metric* = *gram of force = weight of a gram.*
- *Absolute*
  - *English = poundal.*
  - *Metric = dyne.*

A *pound of force* (the force of a pound) is a force equivalent to the attraction of gravity for a pound mass at sea level, 45° N. latitude. A *gram of force* (force of a gram) is equivalent to the attraction of gravity for a gram mass. A *force of a kilogram* is equal to 1,000 grams of force.

A *poundal* is a force that will give to a mass of 1 lb. an acceleration of 1 ft. per sec. per sec. A *dyne* is a force that will give to a mass of 1 gram an acceleration of 1 cm per sec. per sec.

The relation of gravitational to absolute units is:

1 *pound of force* = 32 *poundals*
1 *gram of force* = 980 *dynes*

20. Weight.—The term "weight" is used in two entirely different senses. It may refer either to an *object*, or to a *force*. For example, we say, "Lift the weight from the floor to the table," or "Put the weight on the scale pan," using the term in both cases to refer to an object. On the other hand we may speak of the weight of a body, meaning thereby the force by which it is attracted to the earth. In this sense *weight is a force*. When we say that a stone weighs 10 lb., we mean that it is attracted to the earth by a force of 10 lb.

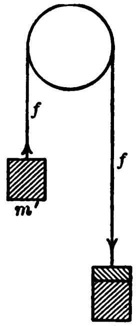


FIG. 15.—Motion of connected bodies in vertical direction.

Example.—In general when the equations $F = ma$, or $W = mg$ are used F or W, as the case may be, is expressed in absolute units. A force of 10 grams will impart what acceleration to a mass of 5 grams?

Solution.—Since we desire to use the equation $F = ma$, *the force must be expressed in absolute units. A force of* 10 *grams* = 9,800 *dynes. Then from* $F = ma$, *we write* $9{,}800 = 5a$, and $a = 1{,}960$ *cm/sec./sec.*

Example.—A force of 10 lb. will impart to what mass an acceleration of 10 ft. per sec. per sec.?

Solution.—A force of 10 *lb.* = 320 *poundals. Then* 320 = *m* × 10, *and hence m* = 32 *lb.*

21. Motion of Connected Masses.—Consider two masses m and m' to be connected by a flexible cord, hung over a vertical fixed pulley, Fig. 15, as illustrated by the case of the Attwood machine. Let $m' < m$, in which case m' will move upward, and m will move downward. If a be the acceleration of the system, then

$$(m + m')a = mg - m'g = (m - m')g,$$

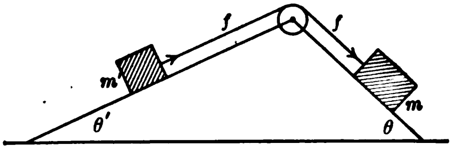


FIG. 16.—Motion of connected bodies on inclines.

If we let f be the force (tension) in the string, we have

$$f = (mg - ma) = (m'g + m'a) = 2gmm'/(m + m'),$$

it being understood that the sign or sense of f depends on whether we consider the body as moving upward or downward.

If the two masses move on an inclined plane, Fig. 16, our equations become

$$(m + m')a = (m \sin \theta - m' \sin \theta')\, g,$$

and

$$f = (mm'g)\,(\sin \theta + \sin \theta')/(m + m').$$

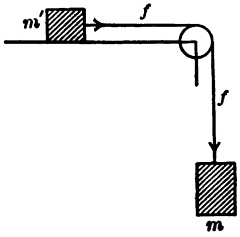


FIG. 17.—Motion of connected bodies at right angles.

If one of the masses move in a vertical direction and the other move in a horizontal direction, Fig. 17, $\sin \theta = 1$, and $\sin \theta' = 0$, and the equation becomes

$$f = mm'g/(m + m').$$

Example.—Given masses m and m' on inclines, as shown in Fig. 16. The angle θ is 60°, and θ' is 30°. The mass of m' is 120 grams. (*a*) Neglecting all friction in the moving system, find what mass m will be required to give an acceleration of 10 cm per sec. per sec. (*b*) Find the pull in the string.

Solutions.—(*a*) $10m + 10 \times 120 = (m \times 0.5 - 120 \times 0.866)\,980$, *whence* $m = 214.7$ *grams and* (*b*) $f = 103{,}046$ *dynes.*

Problems

68. Define gravitational and absolute units of force, in both the English and metric systems.

69. Write dimensional formulæ for (*a*) force; (*b*) pressure.

70. What force in (*a*) dynes; (*b*) grams will give a mass of 10 grams and acceleration of 10 cm. per sec. per sec.?

71. What force in (*a*) poundals; (*b*) pounds will give a mass of 10 lb. and acceleration of 10 ft. per sec. per sec.?

72. A mass of 10 lb. lies on the table. It is acted on by a force (gravitation) that tends to give it an acceleration of 32 ft. per sec. per sec. Find the force which it exerts upon the table in (*a*) pounds; (*b*) poundals.

73. A "ten-pound weight" lies upon the table. Assuming that the term "ten-pound weight" refers to a body having a mass of 10 lb., find the force which it exerts upon the table in (*a*) gravitational units; (*b*) absolute units.

74. A mass of cement exerts a force of 1,000 lb. on a surface 5 by 10 ft. Find the pressure in (*a*) pounds; (*b*) poundals.

75. A force of 1 kg is exerted on a surface 5 by 10 cm. Find the pressure in (*a*) grams; (*b*) dynes.

76. Define: gram mass, pound mass, gram weight, force of a gram, gram of force, pound weight, force of a pound, poundal, dyne.

77. When the mass is given in grams and the acceleration in centimeters per second per second, how is (*a*) F expressed in the equation $F = ma$? (*b*) W, in the equation $W = mg$?

78. When the mass is given in pounds and the acceleration in feet per second per second, in what units is (*a*) F expressed? (*b*) W?

79. A mass of 220.4622 lbs. lies on the floor. Find the force which it exerts upon the floor in (*a*) pounds; (*b*) poundals; (*c*) kilograms; (*d*) dynes.

80. A mass of 10 kg. is acted upon by a force which imparts to it an acceleration of 10 ft. per sec. per sec. Find the force in (*a*) poundals; (*b*) pounds; (*c*) dynes; (*d*) grams.

81. A metal cylinder having a radius of 10 cm, height 20 cm, and density 8 grams per cm^3 rests on one end upon a table. Find the pressure which it exerts upon the table in (*a*) grams; (*b*) dynes.

82. A mass of 10 grams is moving with a velocity of 10 cm per sec. It is then acted upon by a force for 4 sec., after which it has a velocity of 50 cm per sec. Find the force in dynes.

83. A mass of 5 lb. at rest is acted upon by a force for 5 sec., giving it an acceleration of 20 ft. per sec. per sec. Find the force in (*a*) poundals; (*b*) pounds.

84. A force of 20 lb. acting on a mass of 20 lb. will give it what acceleration?

85. What velocity will the body (problem 84) acquire in 5 sec.?

86. A mass of 5 tons is acted upon by a force which imparts to it a change in velocity of 8 ft. per sec. in 4 sec. Find the force in pounds.

87. A given force acts upon a mass of 10 grams for 5 sec. causing the velocity to change from 10 cm per sec. to 30 cm per sec. Find (*a*) the force in dynes; (*b*) in grams.

88. Solve problem 87, substituting pounds for grams and feet for centimeters, giving the result in (*a*) poundals; (*b*) pounds.

89. A mass of 6 kg. is acted upon by a force which imparts to it a change of velocity of 8 m per sec. in 4 sec. Find the force in (*a*) dynes; (*b*) grams.

90. In problem 89 substitute tons and inches for kilograms and meters, and solve for the force in pounds.

91. A mass of 20 grams has an initial velocity of 10 cm per sec. It is acted upon by a force of 120 dynes for 5 sec. Find (*a*) its velocity at the end of 5 sec.; (*b*) the change of velocity; (*c*) the acceleration imparted.

92. A mass of 20 grams is moving with a velocity of 5 cm per sec. It is acted upon by a force for 2 sec. after which it has a velocity of 15 cm per sec. Find the force in (*a*) dynes; (*b*) grams.

93. A force of 20 dynes acting on a mass of 10 grams will impart to it (*a*) what acceleration in 1 sec.? 10 sec.? (*b*) What will be its velocity in 1 sec.? 10 sec.?

94. A force of 1 lb. acting on a mass of 320 lb. will impart to it what velocity in 10 sec.?

95. Given a mass of 100 lb. at sea level. What is its weight in (*a*) pounds; (*b*) poundals? If the body be taken to Denver, Colo., say, how will its mass be affected? its weight?

96. Given a 10-lb. weight (mass of 10 lb.) at sea level. (*a*) What is its weight in pounds? (*b*) If it be carried to a point below sea level, how will its weight be affected?

97. A force of 5 lb. will give a mass of 5 lb. what acceleration?

98. A force of 10 lb. acts for 10 sec. on a body which is free to move, and which has a mass of 10 lb. Find the velocity of the body.

99. A mass of 10 grams is suspended by means of a string. Find the force in dynes exerted on the string (*a*) when the system is at rest; (*b*) when it is drawn upward with a uniform velocity of 10 cm per sec.; (*c*) when it descends with a uniform velocity of 10 cm per sec.; (*d*) ascends with a uniform acceleration of 10 cm per sec. per sec.; (*e*) descends with a uniform acceleration of 10 cm per sec. per sec.; (*f*) descends with an acceleration of 980 cm per sec. per sec.?

100. A mass of 10 lb. is suspended by means of a spring balance from the roof of an elevator. The spring balance is calibrated to give readings in absolute units (poundals). Consider the value of g to be 32 ft. per sec. per sec. What is the reading of the spring balance when the elevator is (*a*) at rest? (*b*) ascending with the uniform velocity of 10 ft. per sec.? (*c*) descending with a uniform velocity of 10 ft. per sec.? (*d*) ascending with a uniform acceleration of 10 ft. per sec. per sec.? (*e*) descending with a uniform acceleration of 10 ft. per sec. per sec.? (*f*) descending with a uniform acceleration of 32 ft. per sec. per sec.?

101. The spring balance (problem 100) is calibrated to give readings in gravitational units (pounds). Find the reading of the balance in each of the cases given in problem 100.

102. A man weighing 160 lb. (gravitational units) stands on the floor of an elevator. What is his weight with reference to the floor of the elevator in poundals and pounds, when the elevator is (*a*) at rest? (*b*) moving with a uniform velocity? (*c*) ascending with a uniform acceleration of 8 ft. per sec. per sec? (*d*) descending with a uniform acceleration of 8 ft. per sec. per sec.?

103. A body having a weight of 1 ton is suspended by means of a rope. The body is pulled upward with an initial acceleration of 4 ft. per sec. per sec. What is the force in pounds sustained by the rope?

104. Two equal masses of 100 grams are hung by a flexible cord over a frictionless pulley. A mass of 10 grams is placed upon one of the hundred-gram masses, Fig. 15. Find (*a*) the acceleration of the system; (*b*) the force exerted on the cord in dynes.

105. A mass of 100 grams hanging by a flexible cord, Fig. 17, drags a mass of 96 grams along the top of a smooth table. Neglecting frictional forces, find (*a*) the acceleration of the system, and (*b*) the stretching force in the cord.

106. A 100-lb. weight is attached to a rope which is wound around a cylinder. The cylinder rotates so that the weight descends with an acceleration of 12 ft. per sec. per sec. What is the stretching force exerted on the rope in (*a*) poundals; (*b*) pounds?

107. A mass of 50 lb. rests upon a smooth horizontal plane. A string fastened to this mass passes over a frictionless pulley and supports vertically a mass of 30 lb. Neglecting frictional forces, find (*a*) the acceleration of the masses; (*b*) the force in the string.

108. A mass of 30 lb. rests upon a smooth plane which is inclined 30° to the horizontal. A string fastened to this mass passes to the top of the plane, over a frictionless pulley, and has a mass of 50 lb. suspended from it. Neglecting friction, find (*a*) the acceleration of the masses; (*b*) the force in the string.

109. A 5-ton safe is drawn up an incline, which makes an angle of 30° with the horizontal. The frictional force is 200 lb. Find the total force required to move the safe.

110. A car having a mass of 40,000 lb. runs up a 1 per cent. grade (tangent of angle = sine). The frictional resistance amounts to 600 lb. What force will move the car up the grade with uniform speed?

CIRCULAR MOTION

22. Centrifugal and Centripetal Forces.—Consider a body of mass m, attached to a string of length r, whirling around in a circle with a uniform velocity about a given point. In accordance with the first law of motion, the body tends at every instant to fly off in a straight line. It is constrained to travel in a circular path by a force, the normal component of which is called the centripetal force. A force equal and opposite to the centripetal force is called the centrifugal force. The centripetal force acts on the body, and is directed from the circumference toward the center of the circle; the centrifugal force is exerted by the body, and is directed from the center toward the circumference.

Centripetal and centrifugal forces may be represented by the equation,

$$F = ma'$$

where F = force in absolute units; m = mass of the body; and a' = acceleration toward the center.

It may be shown that in the case of a body moving with a uniform velocity around a circle, the acceleration a' toward the center is

$$a' = v^2/r = 4\pi^2 r/T^2 = \omega^2 r$$

where v = linear velocity; T = period (time of one revolution); ω = angular velocity.

Problems

111. A body of mass 100 grams moves uniformly around a circle of radius 10 cm $60/\pi$ times per minute. Find (*a*) the period T; (*b*) the linear velocity v in centimeters per second; (*c*) the angular velocity ω; (*d*) the acceleration a'; (*e*) the centrifugal force F in absolute units; (*f*) centrifugal force in gravitational units.

Ans. (*a*) $T = \pi$ sec.; $v = 20$ cm/sec.; (*c*) $\omega = 2$ radians/sec.; (*d*) $a' = 40$ cm/sec./sec.; (*e*) $F = 4{,}000$ dynes $= 4.08$ g.

112. A mass of 2 lb. attached to a string 4 ft. in length is whirled around, making 30 revolutions per minute. Find (*a*) the period T; (*b*) the linear velocity v; (*c*) the angular velocity ω; (*d*) the centrifugal force in absolute units; (*e*) gravitational units.

113. A mass of 32 lb. is attached to a string 2 ft. in length and is whirled around with a uniform velocity, making 15 revolutions per minute. Find (*a*) the centrifugal force in poundals; (*b*) pounds.

114. A body of 2-lb. mass attached to a string 2 ft. in length moves in a circle with a linear velocity of 8 ft. per sec. Find the centripetal force exerted upon the body in (*a*) absolute units; (*b*) gravitational units. (*c*) What is the centrifugal force exerted by the body?

115. Draw a circle to represent the motion of the rotating body of problem 114. Draw XX' and YY' axes passing through the center of the circle. Find the resultant force on the string due to the centrifugal force and the force of gravity, at the points Y, X, Y'.

116. A stone on the end of a string 2 ft. long revolves in a vertical circle. Find the least velocity it could have so that it will maintain its path at the highest point of the circle.

117. A body of 10-lb. mass moves around a circle of 2 ft. radius with a linear velocity of 10 ft. per sec. Find (*a*) the centrifugal force in pounds; (*b*) the centripetal force.

118. A car of mass 40,000 lb. runs around a curve of radius 200 ft. with a speed of 20 ft. per sec. Find the horizontal force in pounds exerted on the outer rail.

119. (*a*) Find in pounds the centrifugal force exerted by a mass of 32 lb. at the equator, the radius of the earth being taken as 4,000 miles. (*b*) What is the weight of this body, if measured by a spring balance?

SIMPLE HARMONIC MOTION

23. Illustration of S.H.M.—A body represented by the light figures *A*, *B*, *C*, etc., moves with a uniform velocity around the circle, Fig. 18. Consider the motion of the projection of this body on an axis of the circle, the *x*-axis, say. The projections of *A*, *B*, *C*, as the body moves around the circle in a positive (counter-clockwise) sense, are represented by the heavy figures *a*, *b*, *c*, and so on.

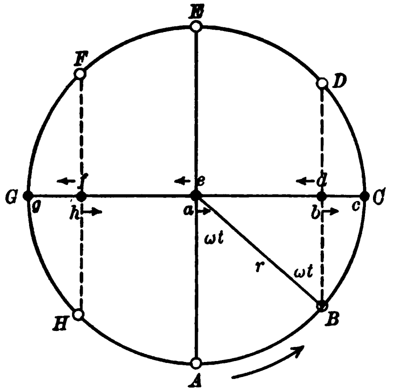


Fig. 18.—Simple harmonic motion and circle of reference.

The vibratory motion of the dark figure on the *x*-axis is an illustration of linear simple harmonic motion.

24. Characteristics of S.H.M.—The main characteristics of the S.H.M., as represented on the *x*-axis, Fig. 18, are as follows: (*a*) The motion is vibratory. (*b*) The body executing S.H.M. has its *maximum velocity* at the middle of its path (at the point *a*), and has zero velocity at the extremities of its path (points *C* and *G*). (*c*) The body has its *maximum acceleration* at the extremities of its path (*C* and *G*), and zero acceleration at the middle point (*a*).

25. Definitions.—(*a*) The circle drawn around the diameter *GC* is called the *circle of reference.* (*b*) The radius of the circle of reference is the *amplitude* of vibration. (*c*) The time required for the body executing S.H.M. to make one complete vibration (that is, the time required for the body on the circle of reference to make one complete revolution) is the *period T.* (*d*) *Phase* is the time which has elapsed since the body executing S.H.M. last passed through the middle point, going in the positive sense. For example, when the body is at *a*, going toward the right, the phase is zero; when it is at *b*, the phase is one-eighth of a period (that is, $T/8$), and the corresponding phase angle (ωt) is *AaB*, in this case 45°. When the body has reached the extremity of its path (that is at *C*), the phase is $T/4$, and the corresponding phase angle is 90°, and so on. The maximum value of the phase angle is, of course, 360°, or 2π radians.

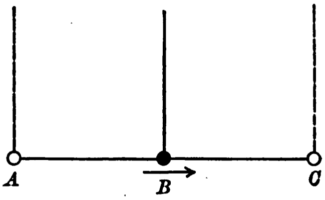


Fig. 19.—S. H. M. of long pendulum.

26. Examples of S.H.M.—(*a*) The motion of the bob of a very long pendulum, Fig. 19, in which the arc *ABC* is practically a straight line is an illustration of S.H.M. In this case the arc *ABC* represents the *x*-axis diameter of the circle of reference. (*b*) The vibration of a body attached to a spiral spring, Fig. 20, is an example of S.H.M. on the *y*-axis. The line *DE* represents the *y*-axis diameter of the circle of reference. (*c*) The vibratory motion of a torsional pendulum, Fig. 21, is an example of angular S.H.M.

27. Equations of S.H.M.—The condition of a body executing S.H.M. is determined at any time t by three defining equations, which determine the displacement x, the velocity V_x, and the acceleration a_x of the moving body. These fundamental equations may be derived geometrically, or by means of the calculus. These equations are, for the x-axis,

$$x = r \sin (\omega t),$$
$$V_x = \omega r \cos (\omega t),$$
$$a_x = - \omega^2 r \sin (\omega t) = - \omega^2 x,$$

in which r = amplitude of vibration; ω = angular velocity ($\omega = 2\pi/T$); t = time which has elapsed since we begin to count time; ωt = phase angle; x = displacement from the middle point in time t; V_x = velocity of the particle in time t; and a_x = acceleration.

The equation $a_x = \omega^2 x$ gives us the basis for the fundamental definition of S.H.M., namely: *Simple Harmonic Motion is a vibratory motion of such a nature that the acceleration is proportional to the displacement.*

It should be noted that in all problems in this text it is understood that we begin to count time when the body is passing through its zero phase, going in the positive sense. In each case then, ωt represents the *phase angle*.

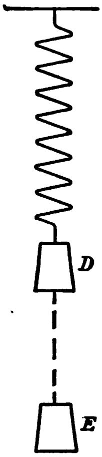


FIG. 20.—S. H. M. of spiral spring.

Example.—Consider Fig. 18 in which a body is executing S.H.M. on the x-axis. At a given instant the body is at the point d, going in a negative sense. The corresponding body on the circle of reference is at D. The period T is 4 sec.; the amplitude r, 2 ft. Find the displacement x, the velocity V_x, and the acceleration a_x, in 1.5 sec. after passing through the middle point, going in the positive sense.

Solution.—In this case $\omega = 2\pi/T = \pi/2$ *radians per second;* $t = 1.5$ *sec. The phase angle* $\omega t = (\pi/2) \times \frac{3}{2} = 3\pi/4$ *radians* $= (\frac{3}{4})\ (180°) = 135°$. *Now* $\sin 135° = \sin 45° = 0.707$; *also* $\cos 45° = 0.707$. *It follows that* (*a*) $x = 2 \times 0.707 = 1.414$ *ft. from middle point;* $V_x = (\pi/2) \times 2 \times 0.707 = 0.707\ \pi$*ft. per sec.;* $a_x = (\pi^2/4) \times 2 \times 0.707 = 0.354\pi^2$ *ft./sec./sec.*

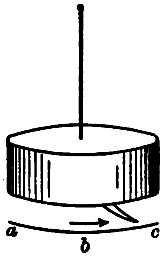


FIG. 21.—Torsional S. H. M.

Problems

120. Consider Fig. 18. What is the phase, and the phase angle when the body executing simple harmonic motion on the x-axis is at a? b? c? d? f? g?

121. Suppose we start to count time when the body is at h, Fig. 18, going in the positive sense. (*a*) What is the time angle when it reaches d? (*b*) What is its phase? (*c*) What is the phase angle?

122. What is the maximum value which the phase angle may have, measured in (*a*) degrees? (*b*) radians? Suppose that we start to count time at *a*, Fig. 18, and the body makes two and a quarter complete vibrations. (*c*) What is the time angle in this case? (*d*) What is the phase angle?

123. A body moves around in a circuit of radius 10 cm with a uniform linear velocity (v) of 20 cm per sec. Find (*a*) the period T; (*b*) the angular velocity; (*c*) the position of the body with reference to the x-axis $1/\pi$ sec. after passing the middle point, in a positive sense.

124. Consider the simple harmonic motion of the projection of this body (problem 123) on an x-axis passing through the center of the circle. Find the displacement x when the phase is (*a*) $T/8$; (*b*) $T/4$; (*c*) $T3/8$; (*d*) $T/2$.

125. Find the displacement x (problem 123) when the phase angle (ωt) is a 30°; (*b*) 45°; (*c*) $\frac{3}{8}\pi$ radians; (*d*) π radians.

126. Find the displacement x (problem 123) (*a*) $\pi/8$ sec. after the particle has passed through the middle point of its path (*a*) going in the positive sense; (*b*) going in the negative sense.

127. Find the velocity V_x, for phase conditions as in problem 124.

128. Find the acceleration a_x, phase conditions as in problem 124.

129. A body attached to a spiral spring, Fig. 20, executes simple harmonic motion. The amplitude of its vibration is 10 cm. Its period T is 4 sec. Counting time from the middle point, find its displacement x in (*a*) 1 sec.; (*b*) 1.5 sec.; (*c*) 2 sec.; (*d*) 3 sec.

130. Find the velocity V_x for conditions as given in problem 129.

131. Find the acceleration a_x, conditions as in problem 129.

132. Find the time required for the body (problem 129) to travel 5 cm from the middle point.

133. An iron weight attached to a spiral spring executes S.H.M. in a vertical direction. The amplitude of vibration is 6 in.; the period, 2 sec. Consider that we begin to count time as the body passes through its middle point going in the positive (upward) sense. Find the displacement x when the time t is (*a*) $\frac{1}{4}$ sec.; (*b*) $\frac{1}{2}$ sec.; (*c*) $\frac{3}{4}$ sec.; (*d*) 1 sec.; (*e*) $\frac{5}{4}$ sec.; (*f*) $\frac{3}{2}$ sec.

134. Find the velocity V_x for each of the cases of problem 133.

135. Find the acceleration a_x for each of the cases of problem 133.

136. Consider that a particle P moves with a uniform velocity around a circle of radius 10 in. with a linear velocity of 5π in. per sec. Find (*a*) the period T; (*b*) the angular velocity ω; (*c*) the time angle in degrees when t is 10 sec.; (*d*) the phase angle in degrees, assuming that we start to count time at the middle point, in positive sense.

137. Consider the S.H.M. of the projection of the particle P (problem 136) on the x-axis. Consider that we count time from the middle point. Find the displacement under the following conditions: (*a*) $t = 1$ sec.; (*b*) 2 sec.; (*c*) 3 sec.; (*d*) 8.5 sec.; (*e*) 10.5 sec.

138. Find the displacement (problem 137) when the phase angle is (*a*) 45°; (*b*) 135°; (*c*) 225°; (*d*) 270°.

139. Find the displacement (problem 137) when the phase angle is (*a*) $\pi/4$ radians; (*b*) $\pi/2$; (*c*) π; (*d*) $\frac{5}{4}\pi$; (*e*) $\frac{3}{2}\pi$.

140. Find the velocity under the conditions of problem 137.

141. Find the acceleration under conditions of problem 137.

142. In a S.H.M. of amplitude 10 in., and period 4 sec., find the displacement, velocity, and acceleration of the body 0.5 sec. after leaving one extremity of its path.

143. In a S.H.M. of amplitude 10 ft. and a period of 20 sec. find the time (t) occupied in traveling 5 ft. from the middle point, going in the positive sense.

144. A particle executing S.H.M. has a period of π sec. Its amplitude is 4 ft. Find its velocity when the phase angle is π radians.

145. Assume that a body O moves from X to X' executing S.H.M. (*a*) At what point does O have its greatest velocity? (*b*) Its greatest acceleration? (*c*) What is its phase when O is at X? (*d*) at X'?

146. In a S.H.M. find the period T when the acceleration at a distance of 0.5176 ft. from the center is 2 ft. per sec. per sec., the amplitude being 2 ft.

147. The bob of a very long pendulum executes S.H.M.; that is, its motion is practically straight-line motion. Its amplitude of vibration is 4 ft.; its period 10 sec. Find the time required for the bob to travel 2 ft. from the middle point, going in the positive sense.

148. Given a S.H.M. of amplitude 8 in., and a period of 4 sec. Find the velocity and acceleration of the body 0.5 sec. after leaving one extremity of its path.

149. A particle having a S.H.M. has a velocity of 4 ft. per sec. when passing through the center of its path. Its period is π sec. (*a*) What is the amplitude of vibration? (*b*) What is the velocity of the particle when its displacement is 1 ft. from the position of rest?

150. A body executing S.H.M. has an acceleration of $\pi^2/4$ ft. per sec. per sec. when the displacement is 1 ft. Find its period.

CHAPTER II

MECHANICS OF SOLIDS (Continued)

WORK AND POWER

28. Work and Energy.—*Work* is the product of force times displacement, the displacement being measured in the direction of the force.

$$Work = W = Fs.$$

Energy is the capacity which a body has for doing work. Energy is of two kinds, potential and kinetic. *Potential energy* (P.E.) is energy of position; *kinetic energy* (K.E.) is energy of motion. Both P.E. and K.E. are measured in terms of work; that is,

$$P.E. = W = Fs,$$
$$K.E. = W = \tfrac{1}{2}mv^2.$$

29. Units of Work.—With reference to the units of force employed, the units of work are of two kinds, namely, *gravitational* and *absolute;* with reference to the system considered, units of work are also of two kinds, *English* and *metric.*

Units of work	*Gravitational*	*English*	= *foot-pound*
		Metric	= *gram-centimeter*
	Absolute	*English*	= *foot-poundal*
		Metric	= *erg.*

The *foot-pound* is the work done by the force of a pound acting through the space of a foot. Foot-pounds = pounds of force × feet. The *gram-centimeter* is the work done by the force of a gram acting through the space of a centimeter. A *kilogrammeter* is the work done by the force of a kilogram acting through the space of a meter. One kilogrammeter = 10^5 gram-centimeters.

The *foot-poundal* is the work done by the force of a poundal acting through the space of a foot. Foot-poundals = poundals × feet. The *erg* is the work done by the force of 1 dyne acting through the space of 1 cm. Ergs = dynes × centimeters. One *joule* = 10^7 ergs.

In the derivation of the equation for kinetic energy, we use the equation $F = ma$, thus expressing F in absolute units. The equation $K.E. = \frac{1}{2}mv^2$, therefore gives results in absolute units, that is, in foot-poundals or ergs.

30. Units of Power.—*Power* is the time rate of doing work. The absolute unit of power is the watt. A *watt* is 10^7 ergs per sec.; that is, 1 joule per sec. According to the Bureau of Standards definition,

One Horsepower (hp.) = 746 *watts.*

According to this definition, 746 watts (1 hp.) are equivalent to 550 ft.-lb. per sec., or 33,000 ft.-lb. per min., at 50° north latitude, and at sea level. In this text, for all problems relating to power, it is assumed that the following equations hold:

$$Watts = \frac{\textit{work in ergs}}{10{,}000{,}000 \times \textit{time in seconds}}$$

$$Hp. = \frac{\textit{work in ft.-lb.}}{33{,}000 \times \textit{time in minutes}} = \frac{\textit{ft.-lb.}}{550 \times \textit{time in seconds}}.$$

Problems

151. Define: foot-pound, foot-poundal, gram-centimeter, kilogrammeter, erg, joule.

152. Write the dimensional formula for work.

153. A force of 100 dynes acts through a space of 100 cm. Find (*a*) the work in ergs; (*b*) joules; (*c*) gram-centimeters.

154. A force of 100 poundals acts through a space of 100 ft. Find the work in (*a*) foot-poundals; (*b*) foot-pounds.

155. A mass of 10 grams moves with a velocity of 10 cm per sec. Find its kinetic energy in (*a*) ergs; (*b*) gram-centimeters.

156. A mass of 10 lb. moves with a velocity of 10 ft. per sec. Find its kinetic energy in (*a*) foot-poundals; (*b*) foot-pounds.

157. A stone having a mass of 20 lb. is carried to the top of a tower 50 ft. in height. (*a*) What potential energy does the stone possess in gravitational units? (*b*) If it be dropped from the top of a tower what kinetic energy in gravitational units will it possess at the instant it strikes the ground?

158. A force of 6 poundals acts through a distance of 5 yd. Find the work done in foot-pounds.

159. Forty-nine joules of work were done by a force acting through a distance of 2 m. Find the force in (*a*) dynes; (*b*) gram-centimeters.

160. In the case of an expanding gas, work may be defined as the product of pressure times volume; that is, $W = pv$. Pressure is force per unit area. Write the dimensional formula for pv.

161. A horse pulls a load weighing 1 ton a distance of 1 mile. If the force exerted upon the traces be 300 lb. what is the work in foot-pounds done by the horse?

162. The lower end of a ladder 30 ft. long stands on the ground at a distance of 6 ft. from the building against which the upper end rests. How much work is done against gravity in carrying 100 lb. to the top of the ladder?

163. A hammer weighing 8 oz. strikes a nail with a velocity of 10 ft. per sec. driving it 1 in. What average force in pounds is exerted by the hammer upon the nail?

164. A mass of 20 grams moves with a velocity of 10 cm per sec. Find its kinetic energy in (*a*) ergs; (*b*) gram-centimeters.

165. A mass of 20 lb. moves with a velocity of 10 ft. per sec. Find its kinetic energy in (*a*) foot-poundals; (*b*) foot-pounds.

166. A body of mass *m* moving with a velocity of 5 ft. per sec. possesses a kinetic energy of 25 ft.-lb. Find the mass of the body.

167. A mass of 196 grams possesses a kinetic energy of 40 gram-centimeters. Find the velocity.

168. A mass of 64 lb. has a velocity of 20 ft. per sec. Find its kinetic energy in foot-pounds.

169. A problem recently given to an engineering class read as follows: "A weight of 10 lb. has a velocity of 10 ft. per sec. What energy does it possess in virtue of its motion?" Solve this problem, assuming that the answer was required in gravitational units.

170. Suppose that the mass *m*, Fig. 22, moves on the incline without friction. The incline *AC* is 10 ft. in length, and makes an angle of 30° with *AB*. The mass *m* is 10 lb. (*a*) If *F* is 5 lb., how many foot-pounds of work will be done in moving *m* from *A* to *C*? (*b*) How many foot-pounds will be required to lift a similar mass from *B* to *C*? (*c*) In this latter case, what force will be required in pounds? (*d*) In poundals?

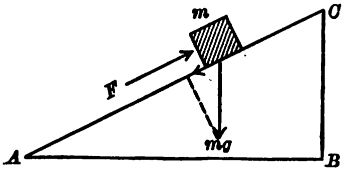


FIG. 22.—Work on inclined plane.

171. What force applied to *m* (problem 170) parallel to *AB* will be required to move *m* from *A* to *C*?

172. Suppose that *AC*, Fig. 22, is 12 ft., angle *BAC* is 30°, and *m* is 20 lb. Find (*a*) the force *mg* in poundals; (*b*) the component of this force down the plane; (*c*) the work in foot-pounds required to move *m* from *A* to *C*; (*d*) the work in foot-pounds required to lift *m* vertically from *B* to *C*.

173. If 1 kg be lifted vertically through a space of 78.74 in. how much work is done in (*a*) kilogrammeters? (*b*) gram-centimeters? (*c*) ergs? (*d*) joules?

174. The ram of a pile driver weighing 20 lb. falls 20 ft. driving the pile downward 2 in. What resistance in pounds does the pile offer?

175. A 10-lb. mass is lifted vertically to a height of 144 ft. Find its potential energy. It is allowed to fall. Find its kinetic energy after it has fallen 144 ft.

176. A mass of 10 lb. falls freely from a point 800 ft. above the ground. (*a*) Find its kinetic energy in foot-pounds after it has fallen for 5 sec.; (*b*) what is its potential energy at the end of the 5 sec.; (*c*) how far has it fallen?

177. A mass of 200 lb. is lifted out of a mine. Find in absolute and gravitational units the work done when the mass is drawn upward a distance of 200 ft. (*a*) with a uniform velocity of 20 ft. per sec.; (*b*) with an acceleration of 4 ft. per sec. per sec.

178. Suppose that the body (problem 177) is drawn upward with a uniform velocity of 20 ft. per sec. for 7.5 sec., and then its motion is retarded for 5 sec. at a rate of 4 ft. per sec. per sec. Find (*a*) in absolute, and (*b*) in gravitational units the work done over the first 150 ft.; the remaining 50 ft.

179. A ton of coal is placed in a car weighing 400 lb. and is hauled out of a mine 200 ft. deep. For the first 5 sec. it ascends with an acceleration of 4 ft. per sec. per sec.; during the second 5 sec. its motion is uniform; during the remaining 5 sec. there is a retardation of 4 ft. per sec. per sec. Find in gravitational units (*a*) the forces used, and (*b*) the work done in getting the car out of the mine.

180. Define: power, watt, kilowatt, horsepower.

181. Write the dimensional formula for power.

182. A mass of 300 kg is lifted vertically through a distance of 50 m in half a minute. Find the power expended (*a*) in watts; (*b*) kilowatts.

183. At what rate (horsepower) is energy expended when a force of 220 lb. is exerted through a distance of 16 ft. in 2 sec.?

184. At what rate (horsepower) is energy expended when a force of 220 poundals is exerted through a distance of 16 ft. in 2 sec.?

185. A mass of 11,190 kg is lifted vertically to a height of 10 m in 10 sec. Find the power expended in (*a*) kilowatts; (*b*) horsepower.

186. Water flows into a mine 500 ft. deep at the rate of 100 cu. ft. per min. Find the horsepower of an engine that will be required to keep the mine dry.

187. In a given steam engine the average pressure exerted on the piston is 180 lb. per sq. in.; the diameter of the piston, 1 ft.; the length of the stroke is 2 ft.; the number of revolutions per minute 120. Find the horsepower.

188. Find the horsepower expended in taking a train of 100 tons up an incline of 1 ft. in 200 ft. at the rate of 20 miles per hr., neglecting friction.

189. A mass of 2204.622 lb. is lifted vertically through a height of 10 m in 10 sec. Find the power expended in (*a*) watts; (*b*) kilowatts; (*c*) horsepower.

190. If a locomotive is rated at 1,200 hp., what force in pounds should it be able to exert while running 40 miles an hour?

191. The area of a piston of a force pump is 200 sq. in., and the length of stroke 20 in. The pump is used to force water into the city mains under a pressure of 60 lb. per sq. in. (*a*) How much work is done per stroke of the piston? (*b*) At what rate (horsepower) will the pump be doing work when it supplies 5,000,000 gallons per 10-hr. day, 1 gallon being equivalent to 231 cu. in.?

192. A train having a mass of 120 tons, including the engine, meets an average resistance on the level of 15 lb. per ton. The engine is of 150 hp. Find the "full speed" of the train; that is, the speed at which the engine exerts a force equal to the resistance.

MACHINES

31. Moment of a Force.—Consider the force F, Fig. 23, to act on the lever arm of length d, thus tending to produce rotation about the point O. The product of the force F times the lever arm d is called the moment of the force, or the *torque*.

Moment of a force = *force* × *lever arm* = Fd,

in which the lever arm d is a straight line measured from the origin O to the line of direction of the force, and at right angles to it. For example, in Fig. 24 suppose that the force F act on the lever arm FO in the direction BF, thus tending to produce rotation about O. The lever arm in this case is the line $BO = d$.

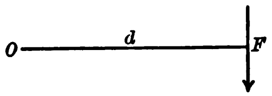


Fig. 23.—Moment of a force, *Fd*.

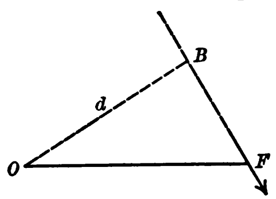


Fig. 24.—Moment of a force, *Fd*.

Two equal parallel forces acting in opposite senses constitute a *couple*. The *moment of a couple* is the product of one of the forces times the perpendicular distance between them; that is, Fd. This being true, it follows that the moment of a couple is the same, no matter where the origin of rotation be chosen in the plane containing the forces.

Example.—Given the couple represented by two equal, parallel, and oppositely directed forces, $F = F'$, Fig. 25, to find the moment.

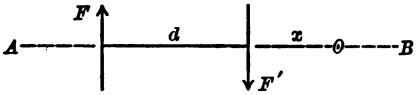


Fig. 25.—Moment of a couple, $Fd = F'd$.

Solution.—Let O be any point on the line AB, and d be the distance between F and F'. The moment tending to produce rotation in a clockwise sense is $+F(d + x)$; *the moment tending to produce rotation in a counter-clockwise sense is* $-F'x = -Fx$. *The moment of the couple is the resultant of these two moments; that is, the moment of the couple* $= +F(d + x) - Fx = Fd$.

32. Solution of Problems by Moments.—Consider Fig. 26. Suppose that the force F acts downward on the bar AB tending to produce rotation in a clockwise sense about the point B. Suppose also that the bar AB is supported by a string AC, in which there is exerted a force F'. Now the moment due to the force F is Fd; the moment due to the force F' is $F'd'$. The system is in equilibrium, therefore these moments are equal in magnitude and opposite in sense, hence we may write

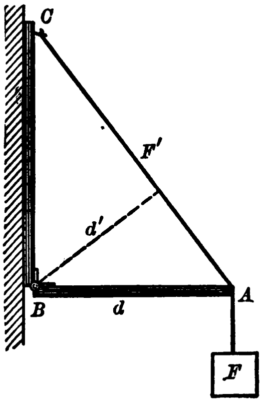


Fig. 26.—Problem of moments.

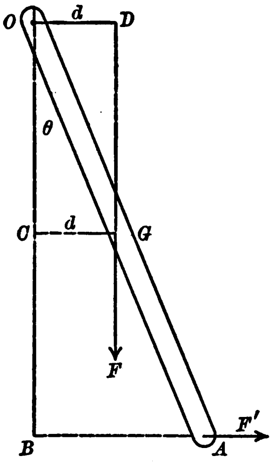


Fig. 27.—Problem of moments.

$$Fd = F'd'.$$

Consider also Fig. 27. Let AO represent a uniform homogeneous bar of length $2l$, the mass of which may be considered to be concentrated at the center of gravity G. The bar is drawn aside so that OA makes an angle θ with OB. The force F represents the weight of the bar and acts downward;

F' is a horizontal force required to hold the bar in position. The equation of equilibrium here is $Fd = F'd'$. But $d = l \sin \theta$, and $d' = 2l \cos \theta$, hence we may write

$$Fl \sin \theta = F'2l \cos \theta.$$

Example.—A uniform metal bar AB, of mass 30 lb., length 8 ft., is supported in a horizontal position, as shown in Fig. 26. A weight of 50 lb. is attached to the end A. We wish to find, by the method of moments, the force exerted on the support AC. The length of BC is 10 ft.

Solution.—We shall first find the value of d', in terms of the angle ACB. The tangent of $ACB = 0.8$, and from our tables we find that $\sin ACB = 0.6247$. Then $d' = 10 \times 0.6247 = 6.247$. The weight of the rod (30 lb.) is considered as if it were concentrated at the center of AB. From the equation of moments ($Fd = F'd'$) we may write $50 \times 8 + 30 \times 4 = F' \times 6.247$, and hence $F' = 83.27$ lb. force.

Example.—Suppose that a rod, of mass 120 lb., and length 12 ft., pivoted at O, Fig. 27, is drawn aside, making with BO an angle of 36°. The force F' acts at right angles to AO. Find (*a*) the force F'; (*b*) the downward force on O; (*c*) the horizontal force on O.

Solution.—(a) $Fd = 120 \times 6 \times 0.5878 = 423.22$. Then, from the equation, $FD = F'd'$, $423.22 = F' \times 12$. Hence $F' = 35.27$ lb. force. (b) The downward force on O = 120 − vertical component of F' = $120 - 35.27 \times \sin 36° = 99.27$ lb. force. (c) The horizontal thrust on O = the horizontal component of F' = $35.27 \cos 36° = 27.53$ lb. force.

Problems

193. Find the moment of a force of 10 lb. applied to one end of a lever arm 10 ft. in length, when the line of direction of the force makes with the lever arm an angle of (*a*) 90°; (*b*) 30°.

194. A horizontal rod AB 100 cm in length is hinged to a vertical support BC, as shown in Fig. 26. A string supporting a mass of 1,000 grams is fastened to the end A, and the other end of the string is fastened to the vertical support at the point C. Find by the method of moments the force (F') exerted on the string BC, in gravitational units, when (*a*) BC is 100 cm; (*b*) 50 cm.

195. Suppose (problem 194) that the cord AC make an angle of 30° with AB, and the weight (1,000 grams) be suspended from the middle point of AB. Find the pull on AC.

196. A uniform rod 12 ft. in length and having a mass of 100 lb. is pivoted at one end and hangs in a vertical position, Fig. 27. The lower end of the rod is now drawn aside until it makes an angle of 30° with the vertical. What force applied to the lower end will be required to hold the bar in this position when the force is applied (*a*) at right angles to the rod; (*b*) at right angles to the vertical.

197. Find the downward force exerted on the pivot under the conditions (*a*) and (*b*) of problem 196.

198. A uniform steel rod *AB*, length 12 ft., mass 200 lb. is pivoted at the end *B* to a vertical support *BC*. The rod *AB* is held in a position at right angles to the support *BC* by means of a rope *AC*, at a point above *B* such that *BC* is 18 ft. Find the force exerted in pounds on the rope.

199. Suppose that the rope *AC* of problem 198 be shortened thus drawing *A* upward, until angle $ACB = 41° 48' 30''$. Make a sketch showing the position of the rod *AB*, and find the force (pull) in the rope.

200. Suppose that the rope (problem 199) be slackened until the bar makes an angle below the horizontal of 60° with the wall. Find the force in the rope.

201. Find the force in the rope (problem 200) when the bar makes an angle of 45° with the wall.

202. A uniform bar 10 ft. in length, mass 5 lb. to the foot, is supported in a horizontal position, as shown in Fig. 26. The angle $ACB = 60°$. Find (*a*) the vertical component at *C*, due to the force along the line *AC*; (*b*) the horizontal component at *C*.

203. A bar *AO*, having a length of 10 ft., a mass of 300 lb., is supported as shown in Fig. 27. The angle *BOA* is 30°. Find (*a*) the force along the line *BA*; (*b*) the downward thrust on the pivot at *O*; (*c*) the horizontal thrust.

204. The bar *AO* (problem 203) is drawn aside by a force acting at *A*, and at right angles to *AO*. The angle *AOB* is 30°. Find (*a*) the force acting at *A*; (*b*) the downward thrust on the pivot at *O*; (*c*) the horizontal thrust.

205. *AB* is a uniform bar of length 8.66 ft., mass 40 lb. It is held in position by a rope *CA*, length 10 ft., which makes an angle of 60° with the wall *BC*. At the point *A* there is suspended a weight *W* of 100 lb. Find (*a*) the force in *CA*; (*b*) the force acting along *AB*, due to the weight of the bar and *W* combined.

206. A uniform metal rod 4 m in length and having a mass of 100 kg. hangs from one end by means of a pivot. A force is applied to the lower end causing the rod to be drawn aside until it makes an angle of 45° with the vertical, the direction of the force being at right angles to the initial position of the rod. Find the force in (*a*) kilograms; (*b*) dynes.

207. A telephone pole standing at the corner of a street carries two systems of wires, which act at right angles to each other, and which produce a resultant force of 2,000 lb. The pole is supported from the top by means of a guy wire which makes an angle of 60° with the ground. The height of the pole is 30 ft. The resultant force on the pole due to the wires lies in the same plane as the supporting guy wire. Find, by the method of moments, the force in the guy wire.

208. The resultant force due to a wire system attached to a telephone pole is 2,500 lb., and is applied at a point 25 ft. from the ground. What force in a supporting guy wire will just counterbalance the force due to the wire system if the guy is attached to the pole 20 ft. from the ground, makes an angle of 30° with the vertical, and lies in the plane of the resultant force?

209. A rigid bar *AB*, 6 ft. in length and having a mass of 400 lb., is supported in a horizontal position by a rod attached to the end *A*, the other end of the rod being fastened to a vertical support at the point *C*, at a distance 8 ft. above the point *B*. A weight of 100 lb. is attached to the bar *AB*. Find the force in the supporting rod when the attached weight is (*a*) at the end of the bar; (*b*) at the middle.

210. Suppose that the rod (problem 209) be increased in length by 2 ft. so that the bar makes an angle with the wall *AC* less than 90°. Find the force in the supporting rod for the two positions mentioned in problem 209.

211. Suppose that the supporting rod be shortened until it is 6 ft. in length. Find the force in the rod for the two positions named in problem 209.

33. Center of Mass.—Suppose that we consider two material particles m and m' at a given distance from each other. Let us select a point p, ($pm = x$ and $pm' = x'$), such that $mx = m'x'$; that is, $mx - m'x' = 0$. The point p is called the center of mass, or centroid of the system. The products mx and $m'x'$ are the moments of mass. The center of mass of a body is a point about which the sum of the moments of mass is zero; that is,

$$\Sigma mx = \Sigma my = \Sigma mz = 0$$

The center of mass, or centroid of the system is sometimes called the *center of inertia*.

Center of gravity. If we consider the gravitational forces which act on the masses of a body m, m', m'', etc., as parallel, then the center of gravity is identical with the center of mass. The center of gravity, then, may be defined as the point through which the total weight of the body, considered as a single vertical force, acts.

For any given plane, it may be shown that the center of mass may be defined by the equation

$$X = \Sigma mx/M,$$

in which X = distance from the plane of reference to the center of mass G; m = any material particle of the system, and x = distance of m from the plane of reference; M = mass of the entire system.

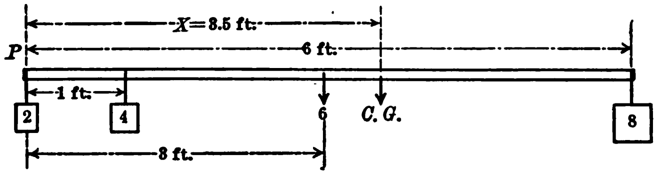


Fig. 28.—Center of gravity.

Example.—Suppose that we have masses m, m', m'', of 2, 4, and 8 lb. respectively, fastened to a uniform homogeneous rod, of length 6 ft., Fig. 28, and wish to find the center of gravity of the system. Mass m is at one end of the rod; m' is 1 ft. from the end; and m'' is at the other end. The mass of the rod is 6 lb., and since the rod is uniform and homogeneous, we may consider that this mass (6 lb.) is concentrated at the midpoint R. We select a plane of reference P at right angles to the line mm'. Now this plane of reference may pass through any point we choose. It is convenient, however, to select one of the end points, as m.

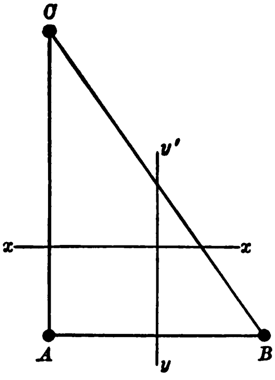


Fig. 29.—Center of gravity.

Solution.—According to our equation the center of mass of the system lies at a distance $X = \Sigma mx/M = (2 \times 0 + 4 \times 1 + 6 \times 3 + 8 \times 6)/(2 + 4 + 6 + 8) = 70/20 = 3.5$. *The center of gravity G, then, lies at a point on mm', to the right of the plane of reference P, equal to 3.5 ft.*

Example—Given three masses of 2, 4, 6, respectively, placed at the vertices of a triangle, Fig. 29, the base of which is 4 ft. and the altitude 6 ft., to find the center of gravity G of the system. In this case we shall choose two planes of reference, one lying in the line AC, and the other in the line AB, and both at right angles to the plane of the paper.

Solution (a).—First let us consider the three bodies with reference to the plane AC. In this case $X = \Sigma mx/M = (4 \times 0 + 2 \times 0 + 6 \times 4)/12 = 2$. *This means that the center of mass with reference to AC lies somewhere on the line yy', and at a distance 2.4 ft. from AC. (b) We must now determine the center of mass with reference to the plane AB. The equation is* $Y = \Sigma my/M = (4 \times 0 + 6 \times 0 + 2 \times 6)/12 = 1$. *The center of mass, therefore, lies at the* *point G, on the intersection of the line xx′ and yy′,* 1 ft. *from AB,* and 2 *ft. from AC.*

General Equations.—If we were to consider a body with reference to its three dimensions it would be necessary to employ, after the manner shown in the last example, all three of the equations,

$$X = \Sigma mx/M, \quad Y = \Sigma my/M, \quad Z = \Sigma mz/M$$

Problems

212. Masses of 10, 20, and 30 grams are placed at the following points on the line *AB*, the length of which is 1 m. The mass 10 is placed at *A;* the mass 20, 60 cm from *A;* and the mass 30, at *B*. Find the center of mass of the system.

213. Suppose that the rod *AB* (problem 212) has a mass of 40 grams. Find the center of inertia (center of mass) of the system.

214. Masses of 10, 20, and 30 grams are placed at the points *A, B, C* of a right triangle. The base *AB* is 15 cm and the altitude *AC* is 12 cm. Find the center of gravity of the system.

215. Suppose that the triangle (problem 214) consists of a board having a mass of 60 grams. Find the center of mass of the system (the board and the masses 10, 20, 30).

216. A bridge having a span of 120 ft. supports a locomotive weighing 60 tons. The center of gravity of the locomotive is 30 ft. from one end of the bridge. The weight of the bridge is 90 tons. Find the force on each pier supporting the bridge.

217. Two parallel forces of 40 and 50 lb. respectively act in the same direction and sense upon a bar at points 9 ft. apart. Find the magnitude and point of application of the resultant.

218. Find the center of gravity of a system consisting of a shaft 8 ft. long, of a mass of 60 lb., and having a 30-lb. pulley at one end, and a 70-lb. pulley at the other end.

219. A uniform metal bar *AB*, 10 ft. in length, having a mass of 2 lb. to the foot, carries a weight of 6 lb. at the end *A*, and a weight of 10 lb., 2 ft. from the end *B*. Where must the bar be supported in order to balance.

220. A right-angled triangular board of uniform thickness has a mass of 2 lb.; height 3 ft.; base 1 ft. Find the center of gravity of the system when a 2-lb. weight is placed (*a*) at the right-angular vertex; (*b*) at the middle of the longest side.

221. A metal beam of variable cross-section is supported by two pillars, one at each end, the load on the pillars being 100 and

200 lb. respectively. When the pillars are shifted so that each stands 1 ft. from the end of the beam, the loads are 90 and 210 lb. respectively. Find the length of the beam.

222. A bar AB, length 8 ft., mass 2 lb., has attached to the end A a mass of 10 lb., and 1 ft. from the end B, a mass of 26 lb. Find the center of inertia of the system.

223. Find the center of inertia of the system (problem 222) when an additional mass of 30 lb. is placed 3 ft. from the end B.

224. A uniform bar AB, having a mass of 2 lb. to the foot, is 10 ft. in length. A 36-lb. weight is placed at the end A; a 6-lb. weight 4 ft. from A; and an 8-lb. weight at the end B. Find the center of gravity of the system.

225. A uniform cubical block 4 ft. on each edge, of 200 lb. mass, is to be overturned about one edge. (*a*) Find the direction and the maximum force required to turn the block most easily. (*b*) What is the total work required to overturn the block?

34. The Law of Machines.—A machine is a device for transferring or transforming energy. In accordance with the law of the conservation of energy, the work done by a force acting on a machine, must equal the work done by the machine. This principle, as applied in mechanics, is expressed by the general law of machines as follows: The force, multiplied by the distance through which it acts, is equal to the resistance overcome, multiplied by the distance through which it acts. This statement of the law takes no account of friction. In terms of Fig. 30, the law may be expressed as

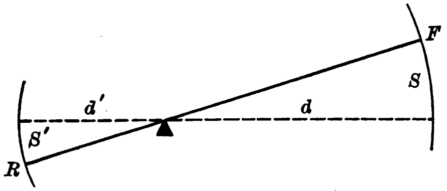


FIG. 30.—Illustrating law of machines.

$$Fs = Rs', \text{ or } Fd = Rd'$$

in which F is the force applied, and d its lever arm; R is the resistance overcome, and d' is its lever arm.

35. Mechanical Advantage.—Three advantages may be derived from the use of a machine. (*a*) We may apply a force in the most advantageous direction, as in the lifting of a weight by means of a pulley. Second, (*b*) we may gain in speed at the expense of force, as in the gearing of a bicycle. And third, (*c*) we may use a small force to overcome a large resistance, as in the case of lifting a heavy weight by means of a lever. When we speak of mechanical advantage we usually refer to the third advantage as mentioned above.

Mechanical advantage is the ratio of the resistance overcome to the force applied; that is, $M.A. = R/F = s/s' = d/d'$.

36. Efficiency of Machines.—The efficiency of a machine is the ratio of the useful work got out of the machine, to the total work put into it. *Efficiency = work out/work in.*

37. Mechanical Powers.—There are six so-called mechanical powers, or simple machines, as follows: The lever, wheel and axle, inclined plane, pulley, wedge, screw. All forms of mechanical machines, however complex, may be reduced in principle to one or more of these simple machines. In general, problems relating to machines may be solved by the use of the equations, $Fs = Rs'$, or $Fd = Rd'$.

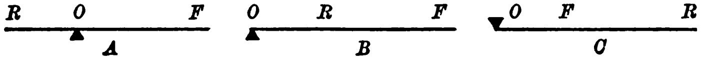


FIG. 31.—Three kinds of levers.

38. The Lever.—A lever is a rigid bar capable of moving about a fixed point called a fulcrum. The three classes of levers are shown in Fig. 31, in which F is the force applied; R the resistance overcome; O the fulcrum; d the force arm; and d' the resistance arm. It should be noted that d is always measured from the fulcrum to the line of direction of the force, and at right angles to it; and the resistance arm d' is measured from the fulcrum to the line of direction of the resistance, and at right angles to it.

The determination of the true weight of a body by the method of double weighing on a balance, the arms of which are of unequal lengths, is a lever problem involving moments, in which we have $Wl = W_1l_1$, and $W_2l = Wl_1$, where l and l_1 are the lengths of the arms; W_1 the apparent weight in one pan, and W_2 the apparent weight in the other; and W is the true weight. From the two equations above we may write $W = \sqrt{W_1W_2}$.

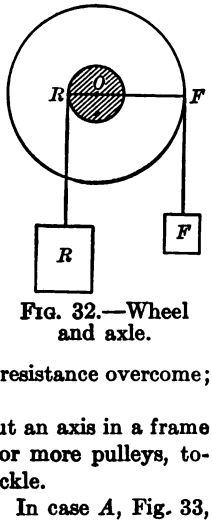


FIG. 32.—Wheel and axle.

39. The Wheel and Axle.—The wheel and axle is an application of the principle of the lever, Fig. 32, in which F may be considered the force applied; R the resistance overcome; OF the force arm d; OR, the resistance arm d'.

40. The Pulley.—The pulley is a wheel turning about an axis in a frame or block. A block, or set of blocks, containing one or more pulleys, together with the attached rope, is called a block and tackle.

The law of machines ($Fs = Rs'$) applies to pulleys. In case A, Fig. 33, when F moves downward 1 ft. R moves upward 1 ft.; that is, $s = s'$. In case B, when F moves 2 ft. R moves 1 ft.; that is, $s = 2$, and $s' = 1$, the mechanical advantage, in this case being $R/F = 2/1$.

In the case of the differential pulley, Fig. 37, we have $Fr = \frac{1}{2}W(r - r')$, where r and r' represent the large and small radii of the fixed pulley system.

41. The Inclined Plane.—The inclined plane is a device for lifting heavy bodies through a vertical height by sliding or rolling them along an incline.

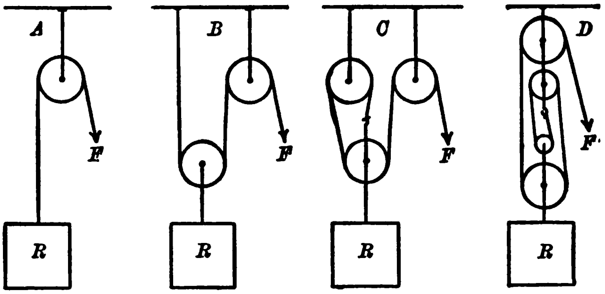


FIG. 33.—Combinations of pulleys.

We shall consider two important cases relating to problems of the inclined plane. First, when the force is applied parallel to the incline, Fig. 34. The equation $Fd = Rd'$ applies. In this case $AC = d$, and $BC = d'$.

Second, when the force is applied parallel to the base, Fig. 35. In this case the force distance d is equal to the length of the base AB, and d' equals BC as before.

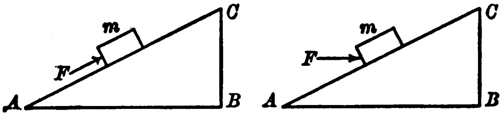


FIG. 34.—Inclined plane.

42. The Wedge.—The wedge is a modified form of the inclined plane. Since the force is usually applied to the wedge by means of a blow from a hammer or a sledge, and the friction factor is very large and cannot be neglected, it is not possible to express a definite relation between the force F and the resistance R as stated in the general law of machines.

43. The Screw.—A screw is a cylinder having a spiral groove cut around its circumference. The spiral ridge is called the thread, and the distance between two consecutive threads, the pitch. The mechanical advantage of the screw is derived from a combination of the principles of the lever and the inclined plane.

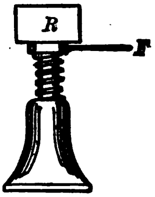


FIG. 35.—Jack screw.

Example.—The lever FR of the jack screw, Fig. 36, is 1 m in length. If one revolution of the force F cause the screw to move upward 1 cm, what resistance R will be overcome by a force of 10 kg.

Solution.—Using the equation $Fs = Rs'$ *we may write,* $2\pi \times 100 \times 10 \times 1{,}000 = R \times 1$. Hence $R = 2{,}000{,}000\pi$ *grams* $= 2{,}000\pi$ *kilograms.*

Problems

226. Make drawings to illustrate the three classes of levers, and explain the mechanical advantage of each.

227. A force of 10 lb. is applied to one end of a lever 10 ft. in length. The resistance is 2 ft. from the fulcrum. Find *R* when (*a*) the fulcrum is 2 ft. from the end *R*; (*b*) when the fulcrum is at the end of the lever (see Fig. 31, *B*). Compare the mechanical advantages of the two systems.

228. A uniform steel rod 10 ft. in length, having a mass of 100 lb., is used as a lever. The resistance arm d' is 2 ft. Find the force required to overcome a resistance of 1,350 lb. when (*a*) the system is used as a lever of the first class; (*b*) second class.

229. Consider Fig. 33. Through what distance must *F* move in order to lift *R* 1 ft. in (*a*) case *C*? (*b*) case *D*?

230. Find the resistance that may be overcome by a force of 10 lb. in each of the cases illustrated in Fig. 33.

231. A force of 10 lb. applied to a system of pulleys as shown in Fig. 36 will support what weight *W*, neglecting the weight of the pulleys.

232. Given an inclined plane, Fig. 34, the base of which is 8 ft.; the height *BC*, 6 ft.; the mass *m* 20 lb. Consider that we may neglect friction between the mass *m* and the smooth incline *AC*. What force applied parallel to the incline will be required to move *m* from *A* to *C*, measured in (*a*) pounds; (*b*) poundals.

233. (*a*) Find the component of the force *mg* down the incline, *CA* (problem 232). (*b*) What force parallel to the base *AB* will counterbalance the component down the incline?

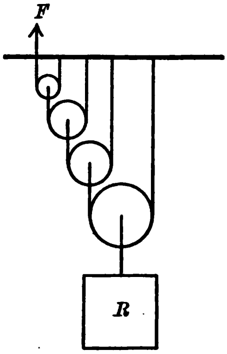


FIG. 36.

234. A mass of 10 lb. rests on a smooth inclined plane, the face of which makes an angle of 30° with the base. Find (*a*) the vertical component of the force in gravitational units due to the mass; (*b*) the component of the force down the plane; (*c*) the component of the force at right angles to the plane.

235. Suppose that the incline of problem 234 be 12 ft. in length. Find what work in foot-pounds will be required to push the mass to the top of the incline, neglecting friction, when (*a*) the force is applied parallel to the incline; (*b*) parallel to the base.

236. Suppose that the frictional force required to slide a box up an incline, the vertical height of which is 4 ft., is 50 lb. The weight of the box is 600 lb. What must be the length of the incline such that a force of 250 lb. applied parallel to the incline may raise the box to the top of the incline?

237. The lever arm of a jack-screw is 3 ft. in length. The distance between consecutive threads is ¼ in. A force of 100 lb. applied to the end of the lever arm will exert what lifting force?

238. The arms of a balance are unequal in length. A given mass weighed in one pan has an apparent mass of 100 grams; in the other pan, 102 grams. (*a*) Find the true mass. (*b*) How does this value compare with the arithmetical mean?

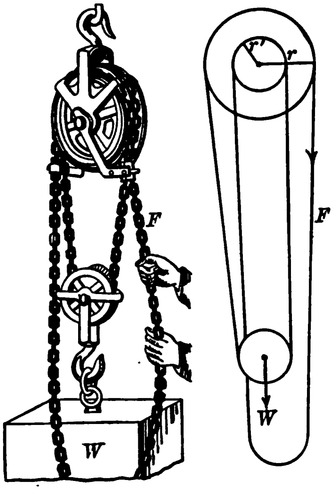


Fig. 37.—The differential pulley.

239. Neglecting friction, what force must be applied to a differential pulley system, Fig. 37, to exert a force of 200 lb. on *W*, the radii of the fixed pulley system being $r = 5$ in., and $r' = 3$ in.?

240. Assume that the efficiency of a differential pulley system is 75 per cent. The radii r and $r' = 4.5$ and 3 in. respectively. A force of 200 lb. at *F*, Fig. 37, will lift what weight (including *W* and the movable pulley)?

MOMENT OF INERTIA

44. Kinetic Energy of Rotation.—Consider a single particle of mass m rotating about a point. The angular velocity of the particle is ω; its linear velocity is $v = \omega r$, in which r is the radius of gyration of the particle m. The kinetic energy of the particle is $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\omega^2 mr^2$.

The term mr^2 is the moment of inertia of the given material particle. The kinetic energy due to the rotation of a system of particles is

$$K.E. = \tfrac{1}{2}\omega^2 \Sigma mr^2.$$

in which Σmr^2 = *moment of inertia* of the system $= I$.

45. Equations, Moments of Inertia.—A few of the more commonly occurring equations for moments of inertia are given below. A more complete list is given in the Appendix, page 194.

Rectangle.—The moment of inertia of a rectangular body of mass M, length a, width b, about an axis through the center of mass, and at right angles to the face, is

$$I = M(a^2 + b^2)/12.$$

Circular Disc.—The moment of inertia of a circular disc, of mass M, radius r, about an axis passing through the center of mass, and at right angles to the face, is

$$I = Mr^2/2.$$

Circular Ring.—The moment of inertia of a circular ring of mass M, outer radius r, inner radius r', about an axis through the center of mass and at right angles to the face, is

$$I = M(r^2 + r'^2)/2.$$

Sphere.—The moment of inertia of a sphere of mass M, and radius r, about an axis through the center, is

$$I = Mr^2 2/5.$$

Example.—A uniform circular disc, having a mass of 200 grams and a radius of 10 cm, rotates about an axis through its center and at right angles to its face, making 15 r.p.m. Find (*a*) its moment of inertia; (*b*) its kinetic energy due to rotation.

Solution.—The moment of inertia of a uniform circular disc about an axis passing through its center of mass is (*a*) $I_o = \frac{1}{2}Mr^2 = 10{,}000\ g\ cm^2$. (*b*) *The period of rotation* $T = 60/15 = 4$ *sec.; and the angular velocity* $\omega = 2\pi/T = \pi/2$ *radians / sec. Hence K.E.* $= \frac{1}{2}I\omega^2 = 1{,}250\pi^2$ *ergs.*

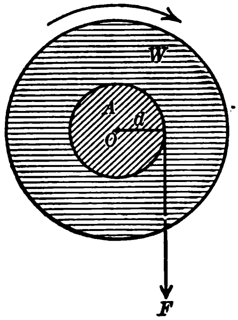


Fig. 38.—Moment of inertia and angular acceleration.

Example.—A rectangular plank, mass 12 lb., length 12 ft., width 18 in. rotates about an axis which is at right angles to its face, and which passes through its center of gravity, making $1/\pi$ r.p.s. Find (*a*) the moment of inertia; (*b*) the kinetic energy of the rotating system.

Solution—The moment of inertia of a rectangular body, under the conditions named, is (*a*) $I_o = M(a^2 + b^2)/12 = 12(144 + 2.25)/12 = 146.25$ *lb. ft.*². *Also,* (*b*), $T = \pi$ *sec., and* $\omega = 2\pi/T = 2$ *radians / sec. Hence K.E.* $= \frac{1}{2}I\omega^2 = 292.5$ *foot-poundals* $= 292.5/32$ *ft.-lb.*

46. Moment of Inertia and Angular Acceleration.—The relation between the *moment of a force* (*Fd*) tending to produce rotation about an axis, the *moment of inertia* (*I*), and the *angular acceleration* (α) of the rotating system, is represented by the equation

$$Fd = \alpha I$$

in which F = the force in *absolute units* tending to produce rotation; d = lever arm, Fig. 38; I = moment of inertia about the axis O; and α = the angular acceleration. In dealing with this equation it is important to recall that the linear acceleration a of any point in a rotating system, where r is the distance of the point from the center of rotation, is

$$a = \alpha r$$

Example.—A string carrying a weight is wrapped around the axle A of a wheel W, Fig. 39. The force F applied to the string is 10 grams; the radius of the axle, 2 cm. The weight moves downward with an acceleration of 20 cm per sec. per sec. Find the moment of inertia of the rotating system.

Solution.—Since F in the equation above is expressed in absolute units, the given force must be reduced to dynes; that is, 10 *grams of force* = 9,800 *dynes. Also, since* $a = \alpha r$, *then* $\alpha = a/r = 20/2 = 10$ *radians / sec. / sec.* $Fd = \alpha I$, *then* $9{,}800 \times 2 = 10I$. *Hence* $I = 1{,}960$ *g cm*^{2}.

47. Moment of Inertia About a Parallel Axis.—Thus far we have considered moments of inertia of systems of symmetrical bodies rotating about axes passing through their centers of mass. It is important, however, to consider the moment of inertia of a body rotating about an axis parallel to the axis through the center of mass. In actual practice we usually have to consider cases of this sort; that is, the moments of inertia of systems rotating about axes other than those passing through their centroids.

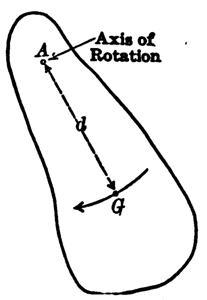


FIG. 39.—Moment of inertia about parallel axes.

It may be shown that the equation for the moment of inertia of such a system is

$$I = I_o + Md^2,$$

in which I = the moment of inertia of the system about the given axis A, Fig. 39; I_o = moment of inertia about an axis parallel to the given axis and passing through the center of gravity G; M = mass of the body; and d = distance between the two parallel axes.

Example.—A circular disc of mass 20 lb., radius 2 ft., rotates about an axis at right angles to its face and passing through a point 18 in. from the center. Find the moment of inertia of the system.

Solution.—$I = I_o + Md^2$. *In this case* $I_o = \frac{1}{2}Mr^2 = \frac{1}{2} \times 20 \times 4 = 40$ *lb. ft.*^{2}; $M = 20$ *lb.*; $d = 1.5$ *ft., hence* $d^2 = 9/4$. *Then* $I = 40 + 20 \times 9/4 = 85$ *lb. ft.*^{2}.

48. The Pendulum.—The period of a pendulum is the time required to make one complete vibration. In the case of a simple pendulum (a material particle suspended by a weightless thread), vibrating under the force of gravity, the period is,

$$T = 2\pi\sqrt{l/g},$$

in which l = distance from the point of suspension to the material particle, and g = acceleration of gravity.

49. The Conical Pendulum.—A simple conical pendulum is a material particle, acted upon by the force of gravity, and constrained to move in a circle of radius r, about a vertical axis AB, Fig. 40. Let g be the acceleration of gravity, and f the centripetal acceleration due to the rotational motion of the pendulum. Then $f = g \tan \theta = gr/h = 4\pi^2 r/T^2$, and therefore

$$T = 2\pi\sqrt{h/g}.$$

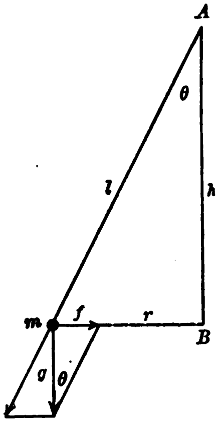


Fig. 40.—Conical pendulum.

50. The Compound Pendulum.—In the equation $T = 2\pi\sqrt{l/g}$, l is the length of an ideal simple pendulum. In the case of a compound pendulum, it is necessary to find the length (l) of an equivalent simple pendulum. This length is

$$l = I/Mh,$$

in which l = the length of an equivalent simple pendulum; I = the moment of inertia of the compound pendulum; M = the mass of the compound pendulum; and h = the distance from the point of suspension to the center of mass of the compound pendulum. The period of a compound pendulum is

$$T = 2\pi\sqrt{I/Mgh}.$$

Example.—A meter stick is suspended at a point 60 cm from the middle and is caused to vibrate. Its mass is 360 grams. Find its period.

Solution.—Since the width of the meter stick is small in comparison with its length, we may consider it as equivalent to a uniform thin rod. $I = I_o + Md^2 = Ml^2/12 + Md^2 = 360 \times 10{,}000/12 + 360 \times 3{,}600 = 1{,}596{,}000$. *Also* $Mgh = 360 \times 980 \times 60 = 21{,}168{,}000$. *Then* $T = 0.5\pi$ *sec.*

51. The Center of Percussion.—The center of percussion of a compound pendulum is a point coincident with the center of oscillation. The center of percussion is that point where a blow, given or received, is most effective and produces the least strain on the support or axis of motion.

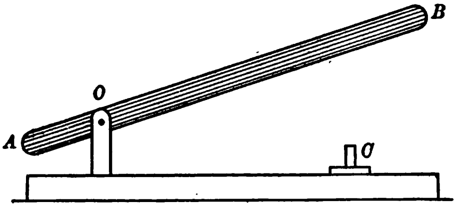


Fig. 41.—Center of percussion apparatus.

Example.—A bar AB, Fig. 41, is pivoted at O so as to strike the body C. AB is 12 ft.; OA is 1 ft., where must C be placed so that when the bar AB falls the pivot O will receive the least strain?

Solution.—The support C must be placed under the center of percussion of AB. The length $OC = l = I/Mh = (Ml^2/12 + Md^2)/Mh = (144/12 + 25)/5 = 7.4$ *ft. from O.*

Problems.

241. Write and explain each term of the following fundamental equations: (*a*) $v = \omega r$; (*b*) $a = \alpha r$; (*c*) $K.E. = \frac{1}{2}I\omega^2$; (*d*) $Fd = \alpha I$; (*e*) $I = I_o + Md^2$.

242. Write the dimensional formula for moment of inertia.

243. Find the moment of inertia of a meter stick, mass 240 grams, about an axis at right angles to its greatest face through (*a*) the middle point; (*b*) one end; (*c*) 20 cm from one end.

244. Find the period of vibration of the meter stick, conditions as given in problem 243.

245. Given a rectangular block, density 10 grams per cm^3, length 40 cm, width 20 cm, thickness 10 cm. Find the moment of inertia of this block about its three axes through the center of mass, and at right angles to the three sets of faces.

246. A circular disc, mass 200 lb., radius 2 ft., makes 120 r.p.m. about an axis at right angles to its greatest face and passing through its center. Find (*a*) its angular velocity ω; (*b*) the linear velocity of a point in the circumference; (*c*) the moment of inertia about its center.

247. Find the kinetic energy of the disc (problem 246) in (*a*) absolute units; (*b*) gravitational units.

248. Suppose that the disc of problem 246 is suspended on an axis passing through its face 6 in. from the circumference. Find the moment of inertia about this axis.

249. Find the period of vibration about the axis conditions as in problem 248.

250. Suppose that the disc rotates about the axis (problem 248) making 30 r.p.m. Find its kinetic energy in foot-pounds due to rotation.

251. Given a uniform board 12 ft. in length and 18 in. in width, and having a mass of 24 lb. Find the moment of inertia of the board (*a*) about an axis through its center of gravity; (*b*) about an axis on a median line and 2 ft. from one end.

252. Find the period of vibration of the board, under the conditions chosen in problem 251.

253. Given a uniform cylindrical disc having a mass of 20 lb., and a radius of 2 ft. Find its moment of inertia about (*a*) its center; (*b*) a point midway between the center and circumference.

254. Find the period of vibration of the disc about the two points mentioned in problem 253.

255. Find the kinetic energy of the disc of problem 253, assuming that it makes 5 revolutions every 10 sec.

256. Suppose that a solid metal wheel having a mass of 100 lb. and a radius of 2 ft. rolls along the ground making 15 r.p.m. Find (*a*) the linear velocity of the center of the wheel with respect to the ground; (*b*) the velocity of a particle at its highest point with respect to the ground; (*c*) the velocity of the particle in contact with the ground, with respect to the ground.

257. The wheel of problem 256 possesses kinetic energy due to its linear velocity ($K.E. = \frac{1}{2}Mv^2$) and also due to its rotation ($K.E. = \frac{1}{2}I_o\,\omega^2$). Find the total kinetic energy due to these two factors.

258. The total kinetic energy of the wheel may be found in another way; that is, by considering its motion with reference to a point in contact with the ground. In this case we use the equation $K.E. = \frac{1}{2}I\omega^2$, in which $I = I_o + Md^2$. Find the kinetic energy of the wheel by this method. How does the result obtained compare with that of problem 257?

259. A given circular disc rolls along a level track with a uniform velocity. At a given instant the linear velocity of a particle at the highest point of the disc is 20 ft. per sec. with respect to the ground. Find (*a*) the velocity of this particle with respect to the center of the disc; (*b*) the linear velocity of the center of the disc; (*c*) the distance the disc will roll in 10 sec.; (*d*) the angular velocity of the disc; (*e*) its period of revolution.

260. A circular disc, mass 100 lb. radius 2 ft., rolls along a level surface. The linear velocity of the center of the disc is π ft. per sec. Find the horizontal velocity with respect to the ground of a particle (*a*) at the highest point in the circumference; (*b*) ½ sec. later; (*c*) 1 sec. later; (*d*) 2 sec. later.

261. Find the kinetic energy of the disc in problem 260 (*a*) due to its linear velocity; (*b*) due to its angular velocity.

262. A metal bar 10 ft. long and having a mass of 100 lb. is pivoted at a point 2 ft. from one end. The free end is raised to a height of several feet and then allowed to fall. Where must a block be placed under the falling bar so that on striking, the pivot will receive the least shock?

263. A rectangular piece of wood of mass 36 lb., is 2 ft. long, 10 in. wide, and 4 in. thick. Find the moment of inertia about axes through its center of mass, and at right angles to its three sets of faces.

264. Given a rectangular piece of board, length *AB* 30 in., width *BC* 10 in. Find its period of vibration when suspended from a point in one edge of the board (*a*) midway between *A* and *B*; (*b*) midway between *B* and *C*.

265. Find the moment of inertia of a disc of mass 1 kg, radius 15 cm, rotating about an axis at right angles to the plane of the disc and at a distance of 5 cm from its circumference.

266. Find the kinetic energy of the disc of problem 265 when its period of rotation is π sec.

267. A metal disc is suspended so as to serve as a torsional pendulum, Fig. 21. A metal ring of mass, 1,000 grams, outer radius 12 cm, inner radius 8 cm, is placed symmetrically upon the disc with respect to its center. How much will the moment of inertia of the system be increased by the addition of the ring?

268. The metal rim of a wheel has a mass of 100 lb. Its moment of inertia is 312.5. lb. ft^2. Its outer radius is 2 ft. Find the inner radius of the rim.

269. A cylindrical projectile of mass 20 lb., radius 6 in., is shot through the air end-on with a linear velocity of 100 ft. per sec. and a rotational velocity equivalent to $20/\pi$ r.p.s. Find its total kinetic energy in foot-pounds.

270. A solid spherical body of radius 4 cm and mass 2 kg has a linear velocity of 20 cm per sec. and an angular velocity of 20 radians per sec. Find its total kinetic energy in joules.

271. A spherical ball having a mass of 200 grams, radius 2 cm, has a linear velocity of 10 cm per sec. and an angular velocity of 10 radians per sec. Find its total kinetic energy in (*a*) ergs; (*b*) gram-centimeters.

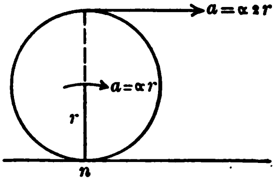


FIG. 42.—Relation of linear acceleration to angular acceleration.

272. A grindstone weighing (having a mass of) 90 lb., and having a radius of 8 in. is supported upon ball bearings so that it turns without appreciable friction. A string is wrapped several times around the stone and a force of 6 lb. is applied to the string, causing the stone to rotate. Find the acceleration of the string.

273. If the grindstone (problem 272) rests on the floor and a force of 6 lb. is applied to the string, Fig. 42, the stone will roll along the floor. Consider the motion of the disc about the point n, in terms of $Fd = \alpha I$. Find (*a*) the linear acceleration of the center of the grindstone; (*b*) the acceleration of the string.

274. Find the kinetic energy of the stone of problem 273, assuming that it has a linear velocity of 10 ft. per sec.

275. A cylindrical disc, Fig. 43, mass 200 lb., radius 18 in., rolls, due to the force of gravity, down an inclined plane CB which makes an angle of 30° with its base AB. Find (*a*) the force in poundals acting on the center of the disc; (*b*) the moment of the force ($Mg \times DE$) tending to rotate the disc about its point of contact D with the incline CB.

276. (*a*) Find the moment of inertia of the disc about its point of contact D with the incline, Fig. 43. (*b*) Find the angular acceleration of the disc as it rolls down the plane.

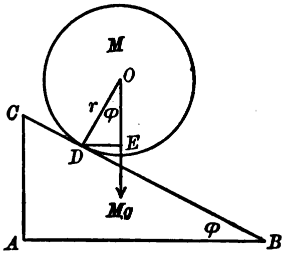


Fig. 43.—Rolling motion on inclined plane.

277. Find (*a*) the linear acceleration of the disc (problem 275) as it rolls down the plane; (*b*) the velocity at the end of 4 sec.; (*c*) the distance it will roll in 4 sec.

278. Let a be the acceleration of a body sliding without friction down an inclined plane, and a' the acceleration of a solid cylinder rolling down the same incline, the motion of both bodies being due to gravity. Suppose that both bodies start from rest, and move for the time t. Show (*a*) that $a' = \frac{2}{3}a$; and (*b*) $v' = \frac{2}{3}v$.

279. Suppose that the two bodies of problem 278 move to the bottom of the plane, a distance s. Let v be the velocity of the sliding body, and v' the linear velocity of the rolling body. Show that at the bottom of the plane $v'^2 = \frac{2}{3}v^2$.

280. Let s be the space traversed by the body sliding down the incline (problem 278) and s' the space traversed by the body rolling down the incline. Show that in time t, $s' = \frac{2}{3}s$.

281. A sphere and a cylinder, both having radii equal to r, start from rest and roll down the same incline for t sec. Let s be the space traversed by the sphere in the time t, and s' the space traversed by the cylinder. Show that $s' = \frac{14}{15}s$.

282. An inclined plane, length 5 m, makes an angle of 30° with the base. A cylinder of mass 2,000 grams radius 10 cm, rolls from the top to the bottom of the plane. Find (*a*) the angular acceleration of the cylinder; (*b*) the linear acceleration (*c*) time required to reach the bottom; (*d*) linear velocity at the bottom; (*e*) angular velocity at the bottom.

283. Find the total K. E. of the cylinder (problem 282) at the instant it reaches the bottom of the plane.

284. The length l of a conical pendulum, Fig. 40, is 30 cm; the mass is 10 grams; and the angle which it makes with the vertical is 30°. Find (*a*) the height h of the cone; (*b*) the period T; (*c*) the angular velocity ω; (*d*) the radius r; and (*e*) the force in the string Am.

285. Find the centrifugal force f exerted by the conical pendulum (problem 284), in gravitational units.

286. A metal rod AB, Fig. 41, of mass 100 lb., and length 14 ft., swings about a pivot at the point O, 2 ft. from the end A. Where must the support C be placed so that AB may strike the most effective blow?

CHAPTER III

MECHANICS OF FLUIDS

FLUIDS AT REST

52. Force Exerted by a Fluid.—Pressure is force per unit area; that is, $P = F/A$. In general, pressure is measured in dynes, or grams per square centimeter, or in poundals or pounds per square inch or square foot. Pressure at a given point in a fluid is equal in all directions.

The force exerted upon any immersed surface, due to the pressure of a fluid, is

$F = PA$; that is,
$F = AHd$, gravitational units, or
$F = AHdg$, absolute units.

In the above equation, A = area pressed upon; H = vertical distance from the surface of the fluid to the center of area (centroid) of the surface pressed upon; d = density of the fluid in grams or pounds per unit of volume; and g = acceleration of gravity.

The *center of area* of a surface is the same as the *centroid* of a surface. The following table gives the centers of area of some common surface areas:

Figure	Center of Area
Parallelogram	intersection of diagonals.
Triangle	intersection of median lines.
Circle	geometrical center of figure.
Spherical shell	center of sphere.
Hemispherical bowl	½ radius, normal to plane surface.
Right cone (hollow)	⅔ distance from vertex to base.

NOTE.—The student must bear in mind that the data given above refer to *surface areas.* For example, while the center of area of a hollow hemisphere is $\frac{1}{2}r$, the centroid of a homogeneous solid hemisphere is $\frac{3}{8}r$, measured from the plane surface; and likewise, while the center of area of a right hollow cone is $\frac{2}{3}h$ from the vertex, the centroid of a homogeneous solid right cone is $\frac{3}{4}h$ from the vertex.

Example.—Given a right conical vessel, the height of which is 6 ft. the radius of the base, 2 ft. The vessel is filled with water, and rests on its base. Find, in gravitational units, the force exerted upon (*a*) the sloping side of the vessel; (*b*) the base.

Solution.—The lateral area A of a cone = ½ *(circumference* × *slant height)* $= 2\pi\sqrt{40} = 12.65\pi$ *sq. ft. Area of base* $= \pi r^2 = 4\pi$ *sq. ft. H for side area* $= \frac{2}{3}h = 4$ *ft. H for base* = 6 *ft. The density d of water* = 62.5 *lb. per cu. ft. Now* $F = AHd$, *and hence* (*a*) *F for the lateral surface* $= 12.65\pi \times 4 \times 62.5 = 3{,}162.5\pi$ *lb.* (*b*) *F on base* $= 4\pi \times 6 \times 62.5 = 1{,}500\,\pi$ *lb.*

53. Pascal's Law.—Pascal's law states that pressure applied to any given area of an enclosed fluid is transmitted undiminished to every like area of the containing vessel. The practical application of this principle is exemplified in the use of the hydraulic press, the hydraulic elevator, and apparatus of similar design.

Example.—The area A of the small piston of a hydraulic press is 4 sq. in.; the radius of the large piston is 1 ft. A force of 10 lb. is applied to the small piston. Find the upward force exerted by the large piston.

Solution.—Let A *be the area of the small piston;* A' *the area of the large piston. Then* $A = 4$ *sq. in.; and* $A' = \pi r^2 = 144\pi$ *sq. in. Let* F *be the force exerted on the small piston, and* F' *that on the large piston. The forces on the two pistons are proportional to the areas; that is* $F : F' = A : A'$. *Hence* $F' = FA'/A = 360\pi$ *lb.*

54. The Total Force Exerted upon an Enclosed Area.—The total force which may be exerted upon any portion of a vessel containing a fluid may be due to two factors: First, the *weight of the fluid* itself; and second, the *force due to the application of an external pressure.* It is evident that the total force upon a given area is equal to the sum of the two factors mentioned above.

Example.—Suppose that a rectangular box 10 cm on each edge is filled with water. Suppose also that a plug having a cross-sectional area of 4 cm^2 is inserted in an opening in the top of the vessel, and that a force of 20 grams is applied to the plug. We wish to find the total force exerted on the six sides of the box, due to the weight of the liquid, and to the force exerted by the plug.

Solution.—Since the force exerted on the plug is given in gravitational units we shall solve for F *in the same units. The force due to the weight of the water* $= F = AHd$. F *on the bottom* $= 10 \times 10 \times 10 \times 1 = 1{,}000$ grams. F *on each side* $= 10 \times 10 \times 5 \times 1 = 500$ *grams, and* F *for the four sides* $= 2{,}000$ grams. F *on the top* $= 0$. *The force, due to the weight of the water, then, on all six sides* $= 3{,}000$ *grams. Now the force on all six sides due to the plug* $= F = PA = (20/4) \times 10 \times 10 \times 6 = 3{,}000$ *grams. The total force due to the weight of the water and that due to the pressure exerted by the plug* 6,000 *grams.*

Problems

287. A rectangular vessel 20 cm long, 10 cm wide, and 10 cm deep is filled with water. Find the force due to the weight of the water on (*a*) the bottom; (*b*) one side; (*c*) one end.

288. Find the total force exerted on the sides, ends, and bottom of the vessel (problem 287) when it is filled with brine, having a density of 1.4 grams per cm^3.

289. Suppose that a tight-fitting cover is placed upon the vessel of problem 288 and that a plug having a cross-sectional area of 5 cm^2 is inserted in the top, and a force of 10 grams is applied to the plug. Find (*a*) the total force exerted on the six faces of the vessel due to the pressure applied by the plug; (*b*) due to the weight of the brine, and the pressure exerted by the plug.

290. A tank 20 ft. long, 10 ft. wide, and 10 ft. deep is filled with water. Find (*a*) the force due to the weight of the water on (*a*) the bottom; (*b*) one side; (*c*) one end.

291. Into the top of the tank (problem 290) there is inserted a plug having a cross-sectional area of 4 sq. in., and upon this plug there is exerted a force of 10 lb. Find the force on the six faces of the tank (*a*) due to the force exerted by the plug; (*b*) due to the weight of the water, and the force exerted by the plug.

292. The radius of the base of a cylindrical vessel is 10 cm, and the height 30 cm. The vessel is filled with water. Find (*a*) the weight of the water in the vessel; (*b*) the force exerted on the bottom; (*c*) the force exerted on the side.

293. A cylindrical plug, radius 2 cm, is inserted into the top of the vessel (problem 292). Upon this plug there is exerted a force of 20 grams. Find the force due to the plug upon (*a*) the bottom of the vessel; (*b*) the side.

294. A cylindrical tank has a radius of 10 ft. and a height of 30 ft. The vessel is filled with water. Find (*a*) the weight of the water in the tank; (*b*) the force exerted on the bottom; (*c*) the force exerted on the side.

295. Into the top of the tank (problem 294) there is inserted a cylindrical plug of radius 6 in., upon which there is exerted a force of 20 lb. Find the force due to the plug on (*a*) the bottom; (*b*) the side.

296. The radius of the base of a right conical vessel is 10 cm, and the vertical height 30 cm. The vessel is filled with water and placed on its base. Find (*a*) the weight of the water in the vessel; (*b*) the force exerted on the bottom.

297. Find the force exerted on the side of the conical vessel (problem 296) when it rests on (*a*) its base; (*b*) its vertex.

298. Three vessels *A*, *B*, *C*, having bottoms of the same area (20 cm on each edge), and sides of the same height (20 cm), are filled with water. The sides of each box are vertical. The ends of *A* are vertical. One end of *B* is vertical, and the other end slopes inward making an angle of 30° with the vertical. One end of *C* is vertical, and the other end slopes outward, making an angle of 30° with the vertical. Make a sketch of the three boxes. Find (*a*) the weight of the water in each box; (*b*) the force exerted on the bottom of each.

299. Find the total force exerted on the two sides of each box (problem 298).

300. Find (*a*) the total horizontal force exerted on the two ends of each box; (*b*) (problem 298) the total normal force on the two ends of each box.

301. A spherical vessel of radius 10 cm is filled with water. Find (*a*) the weight of water in the vessel; (*b*) the total force exerted on the interior of the vessel.

302. The spherical vessel (problem 301) is half full of water. Find (*a*) the weight of water in the vessel; (*b*) the force exerted on the lower half of the vessel.

303. A steel railroad water tank consists of a cylinder of radius 10 ft., height 30 ft. resting upon a hemispherical base. Find (*a*) the weight of water in the tank when it is full; (*b*) the force exerted on the cylindrical sides of the tank.

304. Find the force exerted on the hemispherical surface of the bottom of the tank (problem 303).

305. A thin piece of metal, 20 by 30 cm in area is placed at the bottom of a rectangular tank, 1 m in depth, in such a manner that the 20-cm edge of the metal rests upon the bottom of the tank, and the 30-m edge is vertical. Find the force exerted on one side of the metal.

306. Suppose that the 30-cm edge of the metal (problem 305) makes an angle of 30° with the bottom of the tank. Find the force exerted on the metal.

307. Suppose that the piece of metal (problem 305) is placed at the bottom of the tank in such a manner that one corner touches the bottom, and the diagonal is vertical. Find the force exerted on one side.

308. The radii of the pistons of a hydraulic press are 1 in. and 1 ft. respectively. A force of 100 lb. is applied to the smaller piston. (*a*) Find the force exerted by the larger piston. (*b*) When the larger piston moves 1 in., through what space does the smaller piston move?

309. A rectangular tank, cross-sectional area 4 by 4 ft., height 10 ft. is filled with water. Through the top of the tank there is inserted a plug having a cross-sectional area of 10 sq. in., to which is applied a force of 50 lb. Find the total force in pounds, exerted on (*a*) the bottom of the tank; (*b*) one side; (*c*) top.

310. Compare the force exerted on the bottom of a cylindrical tank filled with water, cross-sectional area 15.708 sq. ft., height 20 ft., with that exerted on the bottom of another cylindrical tank, of radius 5 ft., and height 5 ft., into the top of which is fixed a pipe of cross-sectional area of 1 sq. in., height 15 ft., also filled with water.

311. A vessel having the shape of a right triangular prism, 1 m in height, length, and width, is placed in such a position that one rectangular face is horizontal and one vertical, and is filled with water. Find the force in grams exerted on each face of the vessel.

312. A cubical tank 10 ft. on each edge, is filled with a fluid the specific gravity of which is 1.5. A square plug, 5 in. on each edge, is inserted into the top. A force of 100 lb. is exerted upon the plug, which acts without friction. Find the total force exerted on the six faces of the vessel, due to the weight of the water, and the pressure of the plug combined.

313. A right circular cone, height 50 cm, base radius 10 cm, is placed apex down and is filled with water. Find (*a*) the weight of the water in the vessel; (*b*) the vertical force exerted by the water; (*c*) the force exerted normal to the surface of the cone.

314. Consider the base of the cone (problem 313) to be covered with a water-tight lid. The vessel is filled with water and placed base downward. (*a*) Find the force exerted on the base; (*b*) the force exerted on the sides.

315. A hemispherical bowl of radius 2 ft. is filled with water. Find (*a*) the weight of the water in the bowl; (*b*) the vertical force due to the water; (*c*) the force normal to the surface of the bowl.

316. A sphere of radius r is filled with a liquid of density d. Find (*a*) the weight of the liquid; (*b*) the force normal to the entire surface of the sphere.

317. Find the force exerted on (*a*) the lower half of the sphere; (problem 316) (*b*) the upper half.

318. A cylindrical tank, height 3 ft., radius 1 ft., is filled with water. Into the top of this tank there is inserted a vertical pipe 12 ft. in length, which is also filled with water. Find the force exerted on a cylindrical side of the vessel by the water in the pipe, when the cross-section of the pipe is (*a*) 1 sq. in.; (*b*) 2 sq. in.

319. (*a*) What is the total force exerted on the bottom of the

tank (problem 318). (*b*) How does this force compare with that which would be exerted on the bottom of another tank of height 15 ft., and radius 1 ft.?

55. Center of Pressure.—The center of hydrostatic pressure on any immersed surface is the point of application of the resultant of all the elementary hydrostatic pressures against the elements of the surface. If the area pressed upon is a plane surface all the elementary pressures are parallel, and the problem consists in finding the resultant of a system of parallel forces.

Center of pressure may be defined by means of the equation $X = \Sigma fx/\Sigma f$ in which X is the distance from the surface of the liquid, considered as a plane of reference, to the center of pressure of the area acted upon; f is the force acting on an element of area; fx is the moment due to this force.

56. Center of Pressure on a Rectangle.—Suppose that the upper edge of a rectangle lies in the surface of the water. Let W be the width of the rectangle and Y its altitude. It may be shown that C, the center of pressure, lies on a median line and at a distance $\frac{2}{3}$ of Y, measured from the surface.

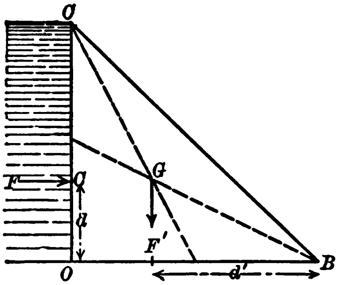


FIG. 44.—Center of pressure.

57. Center of Pressure on a Triangle.—If we have given a triangle the vertex of which lies in the surface of the water it may be shown that $C = \frac{3}{4}Y$.

Example.—A dam whose cross-section is a right-angled triangle 3 m high, is built of stone of density 4. If the water reaches to the top on the vertical side, Fig. 44, what must be the breadth of the base in order that the moment of the force due to gravity about the point B is just equal to the moment of the force due to the water about the point O?

Solution.—Consider unit width of the dam. (*a*) *The area A pressed upon is* 300 *cm*^{2}. $F = AHd = 300 \times 150 \times 1 = 45{,}000$ *grams of force. The point of application of this force, the center of pressure C,* $= \frac{2}{3}Y = 200$ *cm from surface, that is,* 100 *cm from O. The moment of the force about O is, then,* $Fd = 45{,}000 \times 100 = 4{,}500{,}000$. (*b*) *We must now find the moment of the force about the point B, due to the weight of the dam. Let F′ be the weight of the section of the dam under consideration; and let X be the width of the base OB. Then* $F' = (300 \times X \times 4)/2 = 600X$ *grams of force. Since the center of gravity G of the section is at the point of intersection of the medians of the triangle OBC, the line of direction of the force F′ cuts the base OB at a point equal to* $\frac{2}{3}X$, *measured from* $B = d'$. *The moment of the force due to the weight of the dam is, therefore,* $F'd' = 600 \times \frac{2}{3}X^2 = 400X^2$. *According to the conditions of the problem, the two moments Fd and F′d′ are equal in magnitude and opposite in sense, therefore* $4{,}500{,}000 = 600X \times \frac{2}{3}X = 400X^2$. *and hence* $X =$ 106 *cm.*

Problems

320. A right triangular prism, 9 ft. on the base, 6 ft. in height, 10 ft. in length, supports a column of water 6 ft. deep against its vertical face. The density of the prism is 500 lb. per cu. ft. Find (*a*) the moment of the force due to the water tending to overturn the prism; (*b*) the moment of the force due to the weight of the prism tending to hold it in place.

321. Assume that the water presses against the sloping face of the prism (problem 320). Find (*a*) the weight of the water acting vertically downward upon the sloping face. (*b*) Find the force exerted by the water normal to the sloping face. (*c*) The moment of the force with reference to the point *O*, Fig. 44, acting horizontal to the base. (*d*) The moment of the force with reference to *B*, acting at right angles to the base.

322. Suppose that the prism of problem 320 is submerged in water so that its upper edge lies in the surface. Find the moment of the force exerted by the water against the triangular face.

323. The headgate of a flume is 9 ft. in height and 10 ft. in width. It supports, in a vertical position, a column of water 9 ft. in depth. (*a*) Where must a single support be placed so that the headgate will be retained in position? (*b*) What is the moment of the force acting upon the support, tending to turn the headgate about its lowest point?

BUOYANCY OF FLUIDS

58. Archimedes' Principle.—Archimedes' principle, sometimes called the principle of buoyancy, states that a body immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced. A body that sinks in a fluid displaces its own volume; a body that floats displaces its own weight. In both cases the body is buoyed up by a force equal to the weight of the fluid displaced.

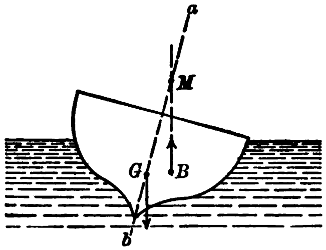


Fig. 45.—Center of gravity below metacenter. Forces *G* and *B* tend to right boat.

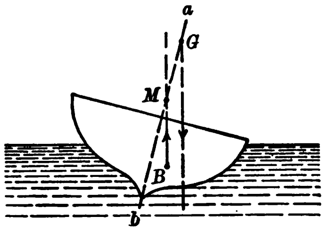


Fig. 46.—Center of gravity above metacenter. Forces *G* and *B* tend to overturn boat.

59. Stability of Floating Bodies.—Let Fig. 45 represent the cross-section of a boat. The point G is the center of gravity of the boat, and B is the center of buoyancy of the water displaced. The *center of buoyancy* of a floating body is the center of volume (centroid) of the space occupied by the liquid displaced. The *metacenter* M is the point of intersection of a vertical line through B with the middle line ab, Fig. 45. At the point G, we may consider that there is a force equal to the weight of the floating body acting *downward;* at B, a force equal to the weight of the water displaced acting *upward.* So long as the metacenter M lies above the center of gravity G, the action of the two forces is to *right the boat.* When, however, M lies below G, the action of the two forces is to *tip the boat over.*

Problems

324. A piece of metal having a mass of 100 grams, and a volume of 10 cm^3 will displace (*a*) what volume of water? (*b*) how many grams of water? (*c*) will be buoyed up by what force? (*d*) will have what weight in water?

325. A cubic foot of lead of mass 700 lb. is immersed in water. (*a*) What volume of water is displaced? (*b*) What weight of water is displaced? (*c*) What buoyant force acts upon it? (*d*) What is its weight in water?

326. A cubic foot of a certain substance weighing 60 lb. is thrown into water. (*a*) Will it sink or float? (*b*) How many pounds of water will it displace? (*c*) What is the buoyant force acting upon it?

327. Make a sketch of Figs. 45 and 46 and explain the action of the two forces, gravity and buoyancy.

328. A cubical block 1 ft. on each edge is immersed in water. Find the buoyant force exerted by the water upon the block (*a*) when the block is just submerged; that is, when its upper face lies in the surface of the water; (*b*) when the upper face is 1 ft. below the surface of the water (*c*) 10 ft. below the surface.

329. A cubical block of wood, mass 250 lb., 2 ft. on each edge, is submerged in a tank of water, so that the upper face of the block lies in and parallel with the surface. The tank is 10 ft. deep. How much work in foot-pounds will be required to force the block to the bottom?

330. Suppose that the block of problem 329 were allowed to float on the surface. (*a*) What part would be submerged? (*b*) What work in foot-pounds would be required to force it to the bottom of the tank?

331. Under standard conditions a cubic foot of hydrogen weighs about 0.006 lb.; a cubic foot of air, about 0.08 lb. A balloon having a capacity of 60,000 cu. ft. is filled with hydrogen. (*a*) Find the buoyant force acting on the balloon due to the air displaced. (*b*) Assuming that the weight of the balloon when collapsed is *W* lb., find the net buoyant force.

332. One of Count Zeppelin's first dirigibles had a capacity of 400,000 cu. ft. What was the maximum net weight which this airship could possess, and still remain in the air, providing it was filled with hydrogen?

60. Density and Specific Gravity.—*Density* D is mass per unit volume; that is, $D = M/V$. *Specific gravity* is the ratio of the density of a given body to that of another substance taken as a standard. Specific gravity may also be defined as the weight of a given volume of a substance divided by the weight of an equal volume of the standard. For solids and liquids, distilled water at 4°C is the standard; for gases, air or hydrogen is chosen as the standard.

Example.—The mass of a cubic centimeter of a given metal is 8 grams. Find (*a*) the density; (*b*) the specific gravity.

Solution.—$D = M/V = 8/1 = 8$ *grams per* cm^3. (*b*) *Sp. g. = weight of body/weight of equal volume of water. One cubic centimeter of water weighs* 1 *gram. Hence sp. g.* = 8/1 = 8.

NOTE.—In metric units, density and specific gravity are numerically equal to each other.

Example.—A cubic foot of a given metal has a mass of 500 lb. Find (*a*) its density; (*b*) specific gravity.

Solution.—$D = M/V = 500/1 = 500$ *lb. per cu. ft.* (*b*) *Sp. g. = weight of body/weight of equal volume of water* = 500/62.5 = 8.

NOTE.—In English units, density is not numerically equal to specific gravity. Densities, however, are now almost universally expressed in terms of metric units, and as such are numerically equal to specific gravities.

61. Methods of Determining Densities.—Some of the more important methods of finding densities are considered briefly in the following paragraphs:

(*a*) *Solids Having Regular Outline.*—Determine mass and volume by direct measurement; find the density by the use of the equation $D = M/V$. See problem 333.

(*b*) *Solids of Irregular Outline.*—Since densities are now almost universally expressed in terms of metric units (grams per cubic centimeter), and also since densities expressed in metric units are numerically equal to specific gravities, we may write *D = sp. g. = weight of body/weight of equal volume of standard.* And since a body immersed in water is buoyed up by the weight of the fluid displaced, that is, by the loss of weight in water, we may also write *D = weight of body/loss of weight in water.* See problem 334.

(*c*) *Solids Lighter than Water.*—In determining the density of a solid of irregular outline, insoluble in but lighter than water, we attach to the body a sinker sufficiently heavy to sink it, and then proceed according to the equation *D = sp. g. = weight of body/loss of weight in water.* See problem 335.

(*d*) ***Density of Solids Soluble in Water.***—In this case we must determine the specific gravity of the solid in some substance in which it is not soluble, and then compute its density in terms of water. See problem 336.

(*e*) ***Density of Liquids.***—The usual method of determining density in the laboratory is that known as the "specific gravity bottle" method. See problem 337.

A second method is to weigh a sinker in water and then in the given liquid. D = *sp. g.* = ***loss of weight in the given liquid/loss of weight in water.*** See problem 338.

A practical method of determining densities of acids, syrups, milk, and other liquids used in large quantities is by use of the hydrometer, the fundamental principle in the employment of which is illustrated in problem 339.

Problems

333. A rectangular block, 20 cm in length, 10 cm in width, and 5 cm in height has a mass of 5 kg. Find its density.

Ans. 5 g/cm^3.

334. What is the density and specific gravity of a body which weighs 10 lb. in air and 8 lb. in water?

Ans. Specific gravity, 5; density, 5 g/cm^3.

335. A sinker which weighs 33 grams in air and 30 grams in water is attached to a piece of wood which weighs 10 grams in air. The wood and sinker weigh 20 grams in water. Find the density of the wood.

Ans. 0.5 g/cm^3.

336. A piece of rock candy weighs 20 grams in air and 10 grams in kerosene, in which it is insoluble. The density of the kerosene is 0.8 gram per cm^3. Find (*a*) the specific gravity of the candy relative to the kerosene, and (*b*) its density with reference to water.

Ans. (*a*) 2; (*b*) 1.6 g/cm^3.

337. Find the density of glycerine from the following data. Weight of bottle, 15 grams; weight of bottle filled with water, 65 grams; weight of bottle filled with glycerine, 78 grams.

Ans. 1.26 g/cm^3.

338. Density of a liquid by the sinker method. A sinker weighs 20 grams in air, 18 grams in water, and 18.36 grams in alcohol. Find the density of the alcohol.

Ans. 0.82 g/cm^3.

339. A given hydrometer consisting of a cylindrical glass rod, closed at both ends, is weighted at one end with mercury, so as to float upright in water. The length of the rod is 60 cm;

its cross-sectional area is 1 cm^2. When placed in pure water 54 cm of the rod were submerged. When placed in sulphuric acid 30 cm were submerged. (*a*) What volume of water was displaced by the hydrometer? (*b*) How many grams of water? (*c*) Mass of the hydrometer? (*d*) How many grams of sulphuric acid were displaced? (*e*) Density of the acid?

Ans. (*e*) 1.8 g/cm^3.

340. A given piece of metal weighs 10 grams in air and 8 grams in water. Find (*a*) the volume of the metal; (*b*) its density; (*c*) specific gravity.

341. Substitute pounds for grams in problem 340 and find (*a*) the volume of the metal in cubic feet; (*b*) its density in pounds per cubic foot; (*c*) its specific gravity; (*d*) its density in grams per cm^3.

342. A cubic foot of a given metal weighs 600 lb. in air. (*a*) What will it weigh in water? (*b*) What is its specific gravity? (*c*) Density?

343. A cubical block 10 cm on each edge has a density of 8. Find (*a*) its weight in air; (*b*) in water; (*c*) in alcohol (density 0.8); (*d*) mercury.

344. A piece of wax having a mass of 10 grams, is attached to a sinker which weighs 20 grams in air and 18 grams in water. The wax and sinker combined weigh 8 grams in water. Find the density of the wax.

345. A piece of wax, density 0.8, is attached to a cubical sinker 2 cm on each edge, density 8. The wax and sinker combined weigh 36 grams in water. Find the volume of the wax.

346. A piece of metal, density 10, contains within its interior a certain cavity. The metal weighs 800 grams in air, and 700 grams in water. Find the volume of the cavity.

347. A piece of metal, mass 100 grams, density 10, is suspended by means of a thread in a beaker containing 500 grams of water. Find (*a*) the force in grams on the string due to the weight of the metal and to the buoyancy of the water combined; (*b*) the force on the bottom of the beaker due to the metal and the water.

348. A cylindrical wooden rod 1 m in length, density 0.8, has fastened to one end a cylinder of brass of the same size as the wood. The density of the brass is 8. The rod is placed in water and allowed to float in a vertical position. Find the length of the brass cylinder such that 10 cm of the wooden rod will float above the surface.

349. A uniform wooden rod is weighted at one end so that when placed in water it floats in a vertical position. The weight of the rod is 100 grams. In water it sinks to a given mark. An additional weight of 20 grams has to be added to the rod in order to cause it to sink to the same mark in a given liquid. Find the density of the given liquid.

350. The density of aluminum is 2.6; the density of silver, 10.6. Compare the weight of a cylinder of aluminum, diameter 1 cm, height 1 cm, with that of a sphere of silver of diameter 1 cm.

351. An irregular piece of metal, of density 8 grams per cm^3, has a cavity within it. The weight of the metal in air is 1,000 grams. When immersed in water it displaces 150 cc. Find the volume of the cavity.

352. Suppose that an alloyed crown of gold and brass weighs 4 kg in air and 3.7 kg in water. The specific gravity of gold is 19.3; that of brass 8.5. Find the amount of gold, and of brass respectively.

352^1. A bottle weighs 2 oz. Filled with water, it weighs 6 oz. Filled with oil, 5.5 oz. Find the density of the oil, and state the denomination of the result.

353. A glass stopper weighs 40 grams in air, 24 grams in water, and 12 grams in sulphuric acid. Find (*a*) the density of the glass; (*b*) the acid.

354. An iron casting weighs 160 lb. in air. There are reasons to suspect that there are blow holes in it. It is, therefore, weighed in water and found to weigh 138.58 lb. Find the volume of the blow holes, assuming that a cubic foot of the iron weighs 480 lb.

355. A balloon when collapsed weighs 100 kg; when filled with hydrogen of density 0.89 gram per liter, it will lift an additional weight of 90 kg. The density of air is 1.293 grams per liter. Find the volume of the balloon when filled.

FLUIDS IN MOTION

62. The Siphon.—The siphon is a device for transferring liquids from a given level to a lower level, over an intervening elevation. It depends for its operation on atmospheric pressure. Fig. 47 shows a siphon in operation. The short arm *ab* of the siphon is measured from the point of application of atmospheric pressure in vessel *A* to the highest point of the siphon; the long arm *ce*, is measured from the highest point to the point of application of atmospheric pressure, as at *e*.

Action of the Siphon.—Let *W* be the pressure of the atmosphere; *w* the downward pressure due to the weight of the water in arm *ab*; *w'* the downward pressure due to the weight of the water in arm *ce*. The upward pressure on both arms is that due to the atmosphere, W. The *effective* upward pressure on the short arm is $W - w$; the effective upward pressure on the long arm is $W - w'$. Since $w' > w$ it follows that $W - w > W - w'$. That is, the greater upward force acts on the short arm, and hence the liquid flows toward the long arm.

Since the pressures (forces per unit area), w and w', are due to the weight of the liquid in the respective arms, $w = hd$ and $w' = h'd$. Now if we let P be the effective upward pressure on the short arm, and P' the effective upward pressure on the long arm, then the *net* pressure producing the flow is

$$P - P' = (W - hd) - (W - h'd).$$

A pressure of one atmosphere = pressure due to 76 *centimeters of mercury =* 30 *inches of mercury =* 1,033.3 *grams per square centimeter =* 1,012,632 *dynes per square centimeter =* 14.7 *pounds per square inch.*

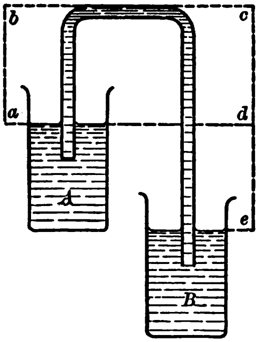


Fig. 47.—Principle of the siphon.

Problems

356. Will a siphon work in a vacuum? Why?

357. Explain (*a*) why the flow ceases when the two arms of a siphon are of the same length; (*b*) why the liquid flows back into the vessel when the outer arm is shorter than the inner arm; (*c*) why increasing the outer arm increases the rate of flow?

358. The specific gravity of kerosene is 0.8; that of sulphuric acid 1.84; mercury, 13.6. Over what height can each of the liquids be siphoned, measured in (*a*) centimeters; (*b*) feet.

359. What is the height of a water barometer when the mercurial barometer reads 76 cm, the density of mercury being given as 13.6? Give results in (*a*) meters; (*b*) feet.

360. Over what height in feet can we siphon brine, the density of which is 1.35, when the barometer reads 72 cm?

361. Consider Fig. 47. Suppose that the siphon tube is of uniform bore and has a cross-section of 1 cm^2. Length of tube *ab*, 20 cm; length of tube *ce* 35 cm; density of liquid 1.5. Find the net effective pressure ($P - P'$) in dynes causing the liquid to flow from the arm *ce*.

362. Consider Fig. 48. Assume that the atmospheric pressure is practically the same at s as at y. The pressure exerted by a cubic inch of the liquid in the tank is 0.01 lb. Find the effective pressure causing the flow when (*a*) the liquid in the tank stands at y; (b) at y'; (e) at y''. (d) Will all the liquid flow out of the tank if s is open? Explain.

63. Velocity of Efflux.—The theoretical velocity with which a liquid issues from an orifice in a vessel is

$$v^2 = 2gh$$

where h = distance from the surface of the liquid to the orifice, and g = acceleration of gravity. The volume of liquid which will flow out of the orifice is

$$V = avt$$

in which V = volume of liquid; a = cross-sectional area of the orifice; v = velocity of efflux; and t = time in seconds. In actual practice this value is never reached. If the opening be a simple orifice without a mouthpiece of any sort, the actual volume of liquid discharged is only about 62 per cent of the theoretical volume. By using a mouthpiece of proper shape and dimensions, the flow may be made about 80 per cent of the theoretical value.

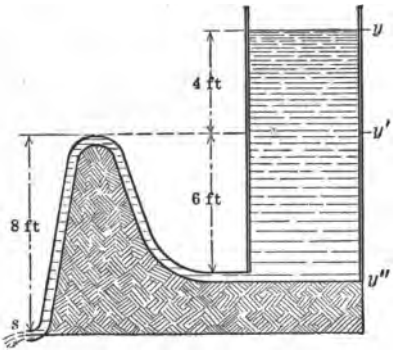


Fig. 48.—Section of a siphon.

64. Velocity of Effusion of Gases.—If a gas of density d, and under a pressure p, be allowed to escape from a vessel through a small orifice with a velocity v, it may be shown that

$$v^2 = 2p/d$$

where v = velocity; p = pressure in absolute units; d = density.

Problems

363. A steel tank is filled with water to a depth of 16 ft. A hole is drilled through the side of the tank, at the bottom, and a short cylindrical tube is fitted to it. The hole in the tube has a cross-sectional area of 1 sq. in. How many cubic feet of water will flow through the tube per minute?

364. The water in a tank is maintained at a constant level 20 ft. from the bottom. Find the total quantity of water that will flow out per hour from two openings of equal cross-sectional area a, one of which is at the bottom of the vessel and the other one halfway down the side, the ratio of the actual volume delivered to the theoretical volume being k.

365. A certain tank is filled with water to a depth of 65 ft. A hole is drilled in the side of this tank 16 ft. from the bottom, which rests on the ground. How far from the base of the tank would the water issuing from this opening strike the ground, provided *v* have its theoretical value and we neglect air friction?

366. The atomic weight of hydrogen is 1; that of oxygen, 16. Compare the velocities with which these two gases will effuse through the walls of a porous vessel, the pressure difference being the same for both.

367. The outer vessel *A* of a diffusion apparatus, contains illuminating gas, of density 0.00081; the porous cup *B* contains air of density 0.001293. (*a*) Compare the rate at which the gas will effuse into the porous cup with that at which the air will effuse outward, the pressure on the two gases being the same at the instant considered. Explain the rise of the liquid in the tube *C*. (*b*) As time goes on what change in the rate of effusion will take place? Explain.

368. If oxygen effuses through a porous vessel at the rate of 500 cc per min. when the pressure difference between the interior and exterior of the vessel is 1 atmosphere, find the rate of effusion of hydrogen when the difference of pressure is 4 atmospheres.

65. Circulatory Motion of the Air.—Few fluid motions are of greater importance to us than those of the air. Especially is this true of what is known to the meteorologist as "cyclones," those immense whirls of air, many miles in diameter, which give rise to the varying conditions of the "weather." The air being heated at a given point gives rise to a diminished barometric pressure, known as a *low* region, Fig. 49. The cold heavy air rushing into this low region, together with the motion imparted due to the rotation of the earth on its axis, gives rise to the cyclonic storms which pass periodically over the country from west to east. In the northern hemisphere these wind storms always rotate in a counter-clockwise sense.

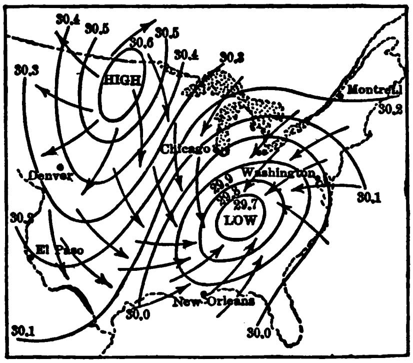


FIG. 49.—Weather map showing rotary motion of the air.

An explanation of this counter-clockwise motion is involved in the solution of the problems which follow.

Problems

369. Consider the three points *a*, *t*, *c*, on a meridian of the hemisphere, Fig. 50, which rotates about a vertical axis through *P* in the sense indicated by the arrow. Suppose that *t* is a target, and that guns situated at *a* and *c* are aimed directly at its center. Considering the motion of the sphere, we wish to find where the balls fired from the guns will strike the target, with reference to its center. All three points move in the same direction and sense, but with different velocities. (*a*) How does the velocity of *c* compare with that of *t*? (*b*) How does the velocity of *t* compare with that of *a*? (*c*) Will the ball fired from *c* strike the target to the right or left of the center, looking toward *P*? (*d*) Where will the ball from *a* strike the target, with reference to its center? (*e*) If the target were free to move about its center, in what sense would it tend to rotate, due to the action of the two balls?

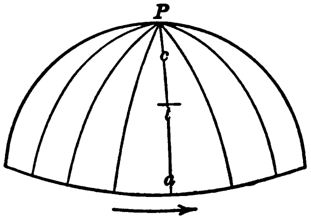


FIG. 50.—Explanation of cyclonic motion of air.

370. Explain why the air rushing into the low area, Fig. 51, tends to set up a circular (cyclonic) motion in a counter-clockwise sense.

371. A person stands with his back to the wind, Fig. 49. On which hand (right hand or left) is the storm center? Why?

372. Suppose that a gun situated at the north pole of the earth is aimed along a meridian at the center of a target, distant 1,200 ft. from the pole. The linear velocity of the target, due to the rotation of the earth is approximately 1 in. per sec.; the velocity of a ball fired from the gun is 1,500 ft. per sec. Where will the ball strike the target with reference to its center?

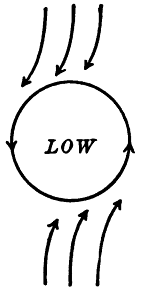


FIG. 51.—Circular motion of the air in "low" area.

373. If the position of the gun and target (problem 372) be exchanged, where will the ball strike the target?

66. Flow of Liquids in Pipes of Variable Section.—Consider water flowing in a pipe of variable cross-section, as shown in Fig. 52. It may be shown mathematically, as well as experimentally, that as the velocity approaches a maximum in the constricted portion of the tube, the pressure approaches a minimum; that is,

$$p < p', \text{ and } p < p''.$$

This means that when a fluid in motion is constrained at any point to increase its velocity, the lateral pressure at that point in the fluid is diminished. This principle has an important application in the operation of the "Venturi" water meter, a modification of which is employed in measuring the water passing through the great Croton aqueduct, which supplies New York City. This principle may also be used to explain the operation of the jet pump, the atomizer or spraying machine, the "ball and nozzle" experiment, and the "curving" of a pitched baseball.

Problems

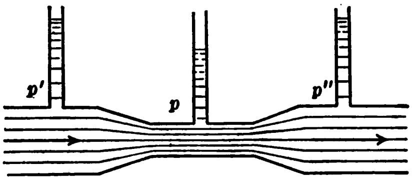


Fig. 52.—Principle of the Venturi water meter.

374. If air be blown through a tube having a flaring lip *ab*, and a card *C* be placed against the lower opening, as shown in Fig. 53, it will be found that the card cannot be blown from the tube, even if the tube be held in a vertical position. Explain this phenomenon in terms of the principle of diminished pressure as illustrated in the case of the flow of water in a constricted pipe, Fig. 52.

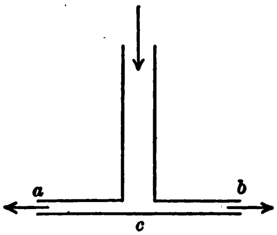


Fig. 53.—Card-and-tube experiment.

375. Consider Fig. 54. Here we have illustrated the various motions of a "pitched" ball. The line *AB* represents the general direction and sense in which the ball is moving at a given instant. *R* is the rotary motion in a clockwise sense; *BC* is the direction and sense in which the ball "curves." We may consider the ball as moving against the air, or, on the other hand, we may consider the air as moving against the ball with a velocity represented by the lines *D* and *E*. As the ball rotates it carries air around with it as shown by the lines *a b c d*. Explain why the ball curves to the right; that is, in the sense *BC*.

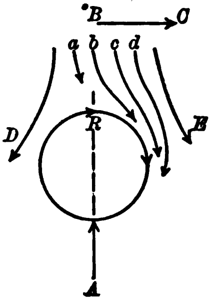


Fig. 54.—Curving of a pitched ball.

CHAPTER IV

MOLECULAR MECHANICS

ELASTICITY

67. Hooke's Law.—*Stress.* When a body is stretched, twisted, or compressed it is said to be distorted. When a force is applied to a medium producing distortion, there is set up in the medium a reaction called a stress. When the system is in equilibrium, the stress is equal to the force per unit area producing the distortion; that is *stress* $= F/a$.

Strain.—When a body is distorted the relative change of configuration of the system is called the strain. In the case of the compression of a gas, the strain is the change of volume per unit of volume; that is *strain* $= v/V$. In the case of the distortion due to a stretching force, the strain is the change of length per unit of length; that is strain $= l/L$.

Hooke's law states that *stress* $= e \times$ *strain*, where e is a proportionality factor known as the *coefficient of elasticity.*

In the case of a change of volume due to a distorting force e is the *coefficient of volume elasticity,* which is

$$C.V.E. = \frac{stress}{strain} = \frac{F/a}{v/V},$$

where F = distorting force; a = cross-sectional area upon which the force acts; v change of volume; and V original volume.

When a rod or wire is stretched the coefficient of elasticity expressed by the ratio of the stress to the strain is called *Young's modulus,* whence

$$Y.M. = \frac{stress}{strain} = \frac{F/a}{l/L}$$

where F = stretching force; a = cross-sectional area of rod or wire; l = change in length; and L = original length.

In a distortion due to twisting, as in the case of a rod fastened at one end and twisted, as shown in Fig. 55, the ratio of the shearing stress to the shearing strain is called the *coefficient of simple rigidity,* or *rigidity coefficient* n, which may be written,

$$n = 2L\mathfrak{T}/\pi\theta r^4,$$

where n = coefficient of simple rigidity; L = length of the rod; $\mathfrak{T}$ = torque, or moment of a force ($F \times$ radius of wheel) producing the distortion; θ = angle of distortion, measured in radians; and r = radius of the rod.

Example.—Given a steel rod AB, 6 ft. in length, radius 0.2 in., clamped at one end, as shown in Fig. 53. The end A is twisted through an angle of 2° by a force of 5 lb., acting on a circumference of the wheel, the radius of which is 3 in. Find the coefficient of simple rigidity n in pounds per inch squared.

Solution.—The length of the rod is 72 *in.; the torque* $\mathfrak{T}$, *in pound inches, is* $5 \times 3 = 15$; $\theta = 2 \times 2\pi/360 = \pi/90$ *radians;* $r^4 = 0.0016$ *in. Then* $n = 2L\mathfrak{T}/\pi\theta r^4 = 2 \times 72 \times 15 \times 90 \times 10{,}000/\pi^2 \times 16 = 1215 \times 10^5/\pi^2 = 12.3 \times 10^6$ *lb./in.*^{2}

68. Determination of Coefficient of Rigidity by Torsional Pendulum.—The torsional pendulum furnishes a method of determining the coefficient of rigidity of a given metal when the latter is in the form of a wire or very thin rod. If a metal bar, or a cylinder, Fig. 21, be suspended by means of a wire, it may be made to oscillate in a horizontal plane with the period T, such that

$$T = 2\pi\sqrt{2LI/\pi n r^4}$$

where L = length of the supporting wire or thin rod· I = moment of inertia of the vibrating bar, or cylinder; n = coefficient of simple rigidity, expressed in absolute units; and r = radius of the wire.

Example.—Suppose that we wish to determine the coefficient of simple rigidity of a piece of metal from which a given sample of wire is made. A cylindrical disc, having a mass of 10 lb., and a radius of 4 in., is suspended from the middle point of one face by means of a piece of the wire whose length is 6 ft., and radius, 0.05 in. The disc makes 26 vibrations per min. Find the coefficient of simple rigidity in pounds per square inch.

Solution.—$T = 60/26 = 2.3$ *sec. The moment of inertia* I *of the disc is* $\frac{1}{2}Mr^2 = 80$; $L = 72$ *in.;* $r^4 = 625/10^8$. *Now from equation* $T = 2\pi\sqrt{2LI/\pi n r^4}$ *we may write* $n = 8\pi IL/r^4T^2 = 8\pi \times 80 \times 72 \times 10^8/625 \times 5.29 = 4{,}378 \times 10^6$ *poundals/in.*^{2} *Now to reduce this value to gravitational units we divide by* $g = 32 \times 12$ *in./sec./sec. That is* $4{,}378 \times 10^6/32 \times 12 = 11.4 \times 10^6$ *lb./in.*^{2}

69. Energy Stored in a Strained Body.—The work required to stretch a body within the limits of its elasticity is stored in the body as potential energy W. This energy is

$$W = \tfrac{1}{2} \times AL \times \textit{stress} \times \textit{strain} = \tfrac{1}{2}Fl,$$

where A = cross-sectional area of the rod; L = length of the rod; F = stretching force; and l = elongation.

Example.—An iron rod, length 5 m, cross-sectional area 0.02 cm^2, is stretched 1 mm by a force of 7 kg. Find the energy in absolute units stored in the rod.

Solution.—$L = 500$ *cm;* $A = 0.02$ *cm*^{2}*; stress = force in dynes/cm*^{2} $= 7{,}000 \times 980 \times 100/2$; *strain* $= l/L = 1/5000$. *Then* $W = (\frac{1}{2} \times 2/100 \times 500 \times 7{,}000 \times 980 \times 50 \times 1)/5000 = 333{,}000$ *ergs. Also,* $W = \frac{1}{2}Fl = \frac{1}{2} \times 7{,}000 \times 980 \times \frac{1}{10} = 333{,}000$ *ergs.*

70. Impact.—The phenomena of impact vary with the masses, the velocities and the elasticities of the colliding bodies. Consider two elastic bodies (two metal balls, say) in which,

m and m' = the masses of the balls,
v and v' = their velocities before impact, and
u and u' = their velocities after impact.

Newton showed that $u - u' = k(v' - v)$, where k is called the coefficient of restitution, a factor depending upon the elasticity of the bodies in collision.

Also, since according to the third law of motion "action is equal to reaction," that is, the change of momentum of one body is equal to change of momentum of the other, we may write $m(v - u) = - m' (v' - u')$.

Now solving for u and u' we have

$$u = (mv + m'v')/(m + m') - km'(v - v')/(m + m'),$$
$$u' = (mv + m'v')/(m + m') + km(v - v')/(m + m').$$

In connection with the above equations it should be noted that the vector quantities v and v' must be taken with their proper signs; that is, if velocity in one sense (to the right, for instance), is considered positive, then velocity in the opposite sense (to the left) will be negative.

We consider that bodies are perfectly elastic when $k = 1$; and perfectly inelastic when $k = 0$.

If a small body m be allowed to fall, striking a large body of mass m' at rest, the velocity of m' will not be appreciably changed, and consequently v' and $u' = 0$. In this case the height of the fall of $m = H$, and $v^2 = 2gH$ downward. The height of the rebound $= h$, and $u^2 = 2gh$ upward. It follows that $k = - v/u = h/H$.

Example.—The coefficient of restitution k between two balls A and B is 0.5. The mass of A is 100 grams, and its velocity v is 20 cm per sec. The mass of B is 50 grams, and its velocity 10 cm per sec. Find the velocity of each ball after impact when (*a*) the balls are moving in opposite senses; (*b*) in the same sense.

Solution.—When the balls are moving in opposite senses, v will be positive, and v' negative. Mass $m = 100$, and $m' = 50$. In case (a) $v = + 20$, and $v' = - 10$. Then $u = + 5$, and $u' = + 20$. This means that both balls after impact move in the same direction and sense, but with different velocities. In case (b) $v = + 20$, and $v' = + 10$. Then $u = + 15$, and $u' = + 20$; that is, both balls move in the same direction and sense.

71. Loss of Kinetic Energy in Impact.—When two bodies collide a certain amount of kinetic energy is lost in the form of heat. The loss of kinetic energy due to impact is

$$\tfrac{1}{2}(1 - k^2)(mm')(v - v')^2/(m + m').$$

Problems

376. A volume of 10 cc of gas under a pressure of 1 atmosphere is enclosed in the short arm of a J-tube by means of mercury. Additional mercury is now poured into the open end of the tube until the volume of the enclosed air is 5 cc. The cross-section of the tube is 1 cm². (*a*) The diminution of volume from 10 to 5 cc was due to what pressure? (*b*) Find the coefficient of volume elasticity of the air. (*c*) How does e in this case compare with the total pressure upon the gas?

377. A mass of gas having a volume of 1,000 cc is under a pressure of 2,000 grams. An additional pressure of 500 grams is applied to the gas, causing the volume to become 800 cc. Find the coefficient of volume elasticity, and compare this value with the total pressure upon the gas.

378. A cylinder having a cross-sectional area of 10 cm^2, and a length of 100 cm, is filled with gas, which is under a pressure of 300 grams per cm.^{2} Into one end of the cylinder there is fitted a piston, which may be considered to move without friction. An additional force of 1 kg is applied to the piston, causing it to move into the cylinder a distance of 25 cm. Find (*a*) the final pressure upon the gas; (*b*) the coefficient of elasticity.

379. A column of water 1 m in length is enclosed in a rigid tube, having a cross-sectional area of 4 cm^2. The coefficient of elasticity of water is 20×10^9 dynes per cm^2. A force of 50 kg applied to the end of the tube will reduce the column of water by what length?

380. A column of water, length 4 m, cross-sectional area 2 cm^2 is reduced in volume by 1.96 cm^3, by a force of 100 kg. Find the coefficient of elasticity of the water in (*a*) gravitational units; (*b*) absolute units.

381. A steel wire 4 m in length, cross-sectional area 0.01 cm^2, is stretched 0.3 mm by a force of 2 kg. Find Young's modulus for this wire (*a*) in grams per cm^2; (*b*) dynes per cm.^{2}

382. A metal rod 12 ft. long, radius 0.1 in., is stretched 0.3 in. by a weight of 300π lb. applied to one end. Find Young's modulus for this rod in pounds per square inch.

383. Young's modulus for a given sample of copper wire is 12×10^{11} dynes per cm^2. The length of the wire is 5 m; its diameter, as determined by a micrometer gage, is 0.6 mm. What force in kilograms will be required to stretch the wire 0.2 cm?

384. A metal rod, length 40 ft., diameter 1 in., whose stretch modulus is 15×10^6 lb. per sq. in., is acted on by a stretching force of 500π lb. Find the elongation in inches.

385. Find what stretching force will be required in the case of the rod of problem 384 to stretch it 0.01 in., if its cross-sectional area be 0.1 sq. in.

386. A rod 1.5 m in length, radius 0.4 cm, is clamped at one end. To the other end there is attached a circular disc of radius 5 cm, Fig. 55. A force of $130^2\pi$ grams causes the disc to rotate through an angle of 4°. Find the coefficient of simple rigidity n for this rod in (*a*) gravitational units; (*b*) absolute units.

387. A metal rod, length 4 ft., radius 0.5 in. is clamped at one end and a circular disc attached to the other end. The radius of the disc is 6 in. A force of 100 lb. applied to the rim of the disc causes a twist of 3°. Find the coefficient of rigidity in pounds per square inch.

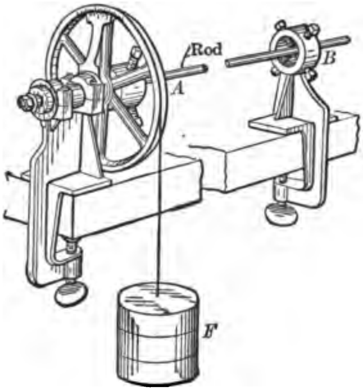


Fig. 55.—Coefficient of rigidity apparatus.

388. A steel rod 60 in. in length and 0.5 in. in diameter is clamped at one end, and has a circular disc of radius 4 in. attached to the other. If the rigidity modulus of this rod (coefficient of rigidity) be 12×10^6 lb. per sq. in., through what angle in radians, will the disc be turned by a force of 20 lb. applied to the rim of disc?

389. A rod 5 m in length, and 4 cm² in cross-sectional area is stretched 0.05 mm by a force of 10 kg. Find (*a*) the stress, in both absolute and gravitational units; (*b*) the strain.

390. A weight of 1 kg attached to a wire having a cross-sectional area of 1 mm², and a length of 5 m, causes it to stretch 0.3 mm. Find Young's modulus in (*a*) gravitational units; (*b*) absolute units.

391. A rod of Bessemer steel 10 ft. long, cross-sectional area 0.05 sq. in. is stretched 0.02 in. by a force of 270 lb. Find Young's modulus (*a*) in gravitational units; (*b*) absolute units.

392. Find the energy in foot-pounds stored in the rod of problem 391 while under strain.

393. What weight in grams will be required to stretch a copper wire 2 mm, the length being 5 m, and the radius 0.1 mm?

394. A brass wire 500 cm long and 1 mm² in cross-section, has a 10-kg mass suspended upon it. How much will the wire be stretched?

395. What would be the stretch of the wire in problem 394 if it were of steel?

396. A wire drawn from Bessemer steel, 4 m in length, 0.01 cm² in cross-section, is stretched by a force of 3 kg. Find the energy in absolute units stored in the wire due to the strain.

397. A stretching force of 8×10^6 dynes will produce what elongation in a rod 1.5 m in length, 5 mm^2 in cross-section, Young's modulus being 12×10^{11} c.g.s. units.

398. A rod having a length of 10 ft. and a radius of 0.2 in. is clamped at one end, and to the other end is attached a circular disc having a radius of 5 in. To the rim of this disc there is applied a force of 10 lb. If the coefficient of rigidity be 10×10^6 lb./in.^{2}, through what angle in degrees will the disc be turned?

399. The rod of problem 398 is clamped at one end, and suspended in a vertical position. To the lower end there is attached a metal cylinder having a diameter of 1 ft. and a mass of 30 lb. Find the period of vibration of this system.

400. A metal cylinder, of radius 10 cm and mass 1 kg, is suspended from a steel wire, of length 1 m and radius 0.07 cm. The period of vibration of the system is 4 sec. Find the coefficient of simple rigidity.

401. An inelastic ball *A*, mass 100 grams, having a velocity of 20 cm per sec., strikes another inelastic ball *B*, of mass 20 grams. Find the velocity of the system under the following conditions: (*a*) When *B* is at rest at the instant of impact; (*b*) when *B* is moving in the opposite sense to *A* with a velocity of 50 cm per sec.; (*c*) with a velocity of 200 cm per sec.; (*d*) when *B* is moving in the same sense as *A* with a velocity of 50 cm per sec.

402. Solve problem 401, assuming the balls *A* and *B* to be perfectly elastic.

403. Solve problem 401 assuming the coefficient of restitution between the two balls *A* and *B* to be 0.5.

404. An ivory ball falls through a height of 1 m, striking a marble slab. The coefficient of restitution between ivory and marble is 0.8. (*a*) To what height will the ball rebound neglecting air friction? (*b*) A second ball dropped upon the slab, rebounds to a height of 70.6 cm. Find the coefficient of restitution between the ball and the slab.

SURFACE PHENOMENA IN LIQUIDS

72. Surface Tension and Energy.—Surface tension is the force exerted in the surface of a liquid, per unit of length of its boundary line; that is,

$$T = F/l.$$

The force F always acts at right angles to the boundary, and is parallel to the surface of the liquid; T is expressed in dynes per unit length.

Suppose that a frame ABC, Fig. 56, be lowered into a given liquid S, and then drawn upward so that a film is stretched across its surface. Now let a light rod D be laid across the surface of the film, and the film below the rod broken. The upper section of the film will contract drawing the rod upward through a distance y. It must be noted that in the case of a free film we have two surfaces to take into account. The entire diminution in surface, therefore, is $2xy$. If E be the surface energy of the film per *unit area*, the potential energy for both surfaces has been decreased by an amount equal to $2Exy$. Now if T be the surface tension per *unit width* of the film, the total surface tension lifting the rod is $2Tx$. The distance moved is y. The work done against gravity is $2Txy$, which is equal to the loss of potential energy, or $2Txy = 2Exy$, and therefore the *surface tension per unit length of the film is numerically equal to its surface energy per unit area.*

For Table of Surface Tensions, see Appendix, page 195.

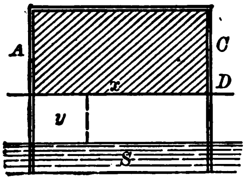


FIG. 56.—Energy in a stretched film.

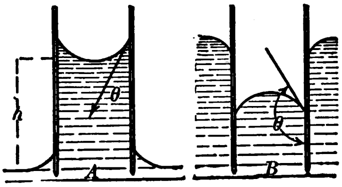


FIG. 57.—Angles of contact in capillary tubes.

73. Capillary Action.—When a capillary tube is thrust into a liquid, the liquid ascends or is depressed in the tube according as its surface is concave or convex, Fig. 57, A and B, the elevation or depression is

$$h = 2T \cos \theta / rdg,$$

where h = elevation or depression of the liquid in centimeters; T = surface tension in dynes per unit length (cm); θ = an angle including the liquid between its surface and the sides of the tube, Fig. 57; r = radius of the tube, in centimeters; d = density of the liquid in grams per cm^3; and g = acceleration of gravity.

In the case of capillary action between two plates

$$h = 2T \cos \theta / udg,$$

where u is the distance between the plates.

74. Normal Pressure on a Curved Film.—A stretched curved film always exhibits a pressure normal to its surface, and directed toward the concave side. It may be shown that for a *cylindrical* film of radius R, the pressure $P = T/R$.

In the case of a *spherical* globule enclosed in a film having *one surface*, as a drop of water, for example, $P = 2T/R$.

In the case of a hollow spherical film (soap bubble) in which there are *two surfaces* of practically equal radii, the pressure is $P = 4T/R$.

Example.—The surface tension of water against air is 75 dynes per cm. A spherical drop of water has a radius of 2 mm. Find (*a*) the pressure normal to the drop; (*b*) the total force acting on the surface of the drop, due to surface tension; (*c*) the potential energy in the surface.

Solution.—(*a*) $P = 2T/R = 2 \times 75/0.2 = 750$ *dynes/cm*^{2}. (*b*) *Area of drop* $= 4\pi r^2 = 0.16\pi$ *cm*^{2}. $F = PA = 750 \times 0.16\pi = 120\pi$ *dynes.* (*c*) *The potential energy E per unit area is numerically equal to the surface tension T; that is,* $E = 75$ *ergs/cm*^{2}. *The total energy in the entire surface of the drop is, therefore,* $EA = 75 \times 0.16\pi = 12\pi$ *ergs.*

75. Angle of Contact.—When a drop of oil is placed on the surface of water, the shape which the drop assumes, Fig. 58, depends upon the relative magnitude of the following surface tensions: T_{oa}, the surface tension between oil and air; T_{ow}, the surface tension between oil and water; T_{wa}, the surface tension between water and air. The drop will be flattened by the tension T_{wa} until the projections of the vectors T_{oa} and T_{ow} is equal to T_{wa}; that is

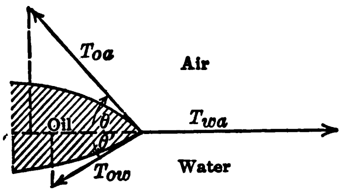


FIG. 58.—Form of drop of oil on water.

$$T_{wa} = T_{oa} \cos \theta + T_{ow} \cos \theta'.$$

Problems

405. In Fig. 56 we have represented a rectangular wire support upon which there is stretched a soap film, of length 5 cm, width 5 cm, the surface tension of which is 36 dynes per cm. (*a*) What is the force exerted upon the rod *D*, due to the surface tension of the film? (*b*) What potential energy does the film possess?

406. What is the height to which water will rise in a capillary tube 0.06 cm in diameter, assuming the angle of contact to be negligible?

407. How much will the surface of mercury be depressed in a glass tube of radius 0.02 cm, the angle of contact being 135°?

408. How high will a liquid of density 0.86 rise in a tube, having a diameter of 0.5 mm, and an angle of contact of 20°, the surface tension being 40?

409. Compute the diameter of a glass tube in which pure water rises to a height of 12 cm, the angle of contact being negligible.

410. A soap bubble of radius 4 cm is made from a solution having a surface tension of 25 dynes per cm. Find (*a*) the pressure normal to the surface of the bubble; (*b*) the force exerted upon the air within the bubble.

411. Two spherical globules of mercury having radii of 1 mm and 2 mm respectively, unite to form one drop. (*a*) How does the surface of the resulting drop compare with the sum of the surfaces of the two drops? (*b*) How does the surface energy of the resulting globule compare with the surface energies of the original drops?

412. The surface tension of a certain soap solution is 30 dynes per cm. What is the force exerted upon the enclosed air in a bubble made from this solution, due to surface tension, the radius of the bubble being 3 cm?

413. Find the angle of contact of turpentine with glass from the following data: Radius of capillary, 0.1 mm; density of turpentine, 8.6; surface tension, 28.8; height to which liquid rises in tube, 6.52 mm.

414. A given sample of mercury, the surface tension of which is 535, is depressed 12.3 mm in a capillary tube of radius 0.05 cm. Find the angle of contact of the mercury with glass.

415. Compute the angle of contact of a liquid *A* with a substance *B* from the following data: Surface tension of liquid, 50 dynes per cm; density of *A*, 0.8; diameter of capillary tube *B*, 0.05 mm; height to which *A* rises in *B*, 5 cm.

416. The surface tension of a given oil against air is 58; against water, 32; that of water against air, 75. If the angle made by the oil and the water with the surface is 20° (Fig. 58), what is the angle which the oil in air makes with the surface?

417. The surface tension of olive oil against air is 36.9 dynes per cm, and against water, 20.7. The surface tension of water is 75. (*a*) How do the surface tensions named above compare with the pull (surface tension) of the water? (*b*) What will happen to the drop of oil if it is placed upon water?

418. The surface tension of grease is greater than that of gasoline. In cleaning a grease spot from cloth, in what direction (inward or outward) will the grease move when the gasoline is placed (*a*) in the center of the spot (*b*) in a ring around the margin of the spot? (*c*) How should gasoline be applied to the spot so as to prevent the grease from spreading into the cloth?

76. Diffusion of Liquids.—If two liquids which are miscible are introduced into a vessel so that the lighter lies above the denser, diffusion will take place, some of the lighter liquid passing down, and some of the denser liquid passing upward. The essential facts relating to diffusion of liquids are: (*a*) The rate of diffusion is very slow. (*b*) In general the rate of diffusion increases with the increase of temperature. (*c*) The rate of diffusion is proportional to the *concentration gradient* c, where c is the change of concentration per unit of length considered. (*d*) The rate of diffusion is proportional, also, to the *diffusion constant* k, where k is the number of grams of the solute which diffuses through unit area (1 cm^2), per unit concentration gradient, per unit time (1 day). The mass m which will diffuse through any given area in the time t is

$$m = k \times c \times s \times t$$

where m = mass in grams; k = diffusion constant; c = gradient constant; s = surface area lying between the two solutions; t = time in days.

Problems

419. A cylindrical vessel having a radius of 4 cm contains a salt solution, above which there is a quantity of pure water. The vertical concentration gradient of the salt solution is 2. The diffusion constant of the salt solution is 0.76. Find the number of grams of salt which will diffuse into the water in 8 hr.

Ans. 6.08π g.

420. (*a*) Consult table of diffusion constants (Appendix, page 196) and compare the rate of diffusion of hydrochloric acid with that of common salt (NaCl). (*b*) If hydrochloric acid were placed in the vessel of problem 419 instead of a salt solution, how much acid would diffuse upward in 1.5 days?

421. From the following data compute the diffusion constant for cane sugar. It was found that 3.1 grams of sugar from a solution the concentration gradient of which was unity, passed through 2 cm^2 of surface in 48 hr. Find k.

422. The diffusion constant of sodium chloride at 10°C is 0.91. It was shown experimentally that 9.1 grams of the salt passed through a surface of 4 cm^2 in 2 days. Find the concentration gradient.

423. Compare the diffusion constant of albumen with that of sodium chloride, and compute the number of grams of albumen which will diffuse, under the conditions of problem 422.

424. What time in days will be required for 10.44π grams of hydrochloric acid, possessing unit concentration gradient, to diffuse through a circular area having a diameter of 4 cm, the temperature being 5°C?

77. Diffusion through Membranes. Osmosis.—Liquids diffuse readily through certain membranes. The diffusion of substances through such septa is called osmosis. The separation of crystalloids from colloids by means of an intervening membrane is called dialysis. If an aqueous solution of a salt or of sugar be separated from water by means of a membrane, Fig. 59, which allows osmosis to take place from W to S more readily than from S to W, the solution rises in the tube giving rise to a pressure equivalent to hdg, which measures the osmotic pressure of the solution in S.

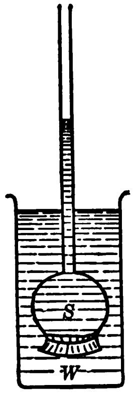


Fig. 59.—Osmotic-pressure apparatus.

Osmotic pressure varies (*a*) as the concentration c of the solution, and (*b*) as the absolute temperature T; it also depends upon (*c*) the kind of membrane, and (*d*) the nature of the solutions separated by it.

Example. Pfeffer's Experiment.—Pfeffer performed a series of classic experiments in which he measured the osmotic pressure of sugar solutions, using an apparatus as shown in Fig. 60. The osmotic pressure of the solution in S was measured by means of the mercury manometer M. He found that a concentration of 20.16 grams of sugar per liter of water gave an osmotic pressure equivalent to 101.6 cm of mercury. In another experiment, at the same temperature, he found that 61.19 grams of sugar dissolved in 1 liter of water gave a pressure of 307.5 cm. (*a*) Compare the ratio of the osmotic pressures to that of concentrations. (*b*) Find the volume of solution in each case containing 1 gram molecule of sugar, the molecular weight of sugar being 342. (*c*) Find the product of pressure and volume (pv) in each case.

Solution.—(*a*) $P : P' = 101.6 : 307.5 = C : C' = 20.16 : 61.19 = 1 : 3$ (*nearly*). (*b*) *Since there are* 20.16 *grams in* 1,000 *cc of solution,* 342 *grams will require* (342/20.16) × 1,000 = 16,964 *cc; and* (342/61.19) × 1,000 = 5,589 *cc.* (*c*) $pv = 101.6 \times 16{,}964 = 1{,}723{,}500$; *and* $307.5 \times 5{,}589 = 1{,}718{,}600$.

Example.—Van't Hoff's application of Pfeffer's data. The gas law states that the product of pressure and volume is equal to the absolute temperature times a constant R; that is $pv = RT$. Van't Hoff showed by means of Pfeffer's data that a dilute solution of sugar in water obeys the same general law as a gas. Selecting the first set of data given in the example above, find the value of R, the temperature being 15°C.

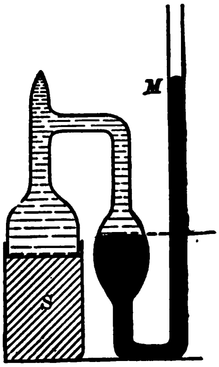


Fig. 60.—Pfeffer's osmotic-pressure apparatus.

Solution.—The pressure $p = 102 \times 13.6 \times 980 = 1359 \times 10^3$ *dynes/cm*^{2}. *The volume v containing a gram molecule of the solute* = 16,964 *cc. The absolute temperature* $T = 15 + 273 = 288$. *Then* $R = 1359 \times 10^3 \times 16{,}964/288 = 8 \times 10^7$.

How does the value of R thus obtained compare with that obtained from the equation when applied to a gas? See page 94.

Problems

425. Pfeffer found that a certain concentration of sugar gave a pressure of 53.5 cm of mercury, the product of pv expressed in cm and cc being 1,822,000. Find the concentration in grams per liter.

Ans. 10.04 grams/liter.

426. If the temperature at which the experiment referred to in problem 425 was performed was 20°C, what is the value of *R*, as computed from the above data?

427. At a given temperature, a 1 per cent. solution of sugar in water (10 grams per liter) gave an osmotic pressure equivalent to 50.5 cm of mercury. The computed value for *R* was 82,202,000. What was the temperature at the time of the experiment?

428. Sugar does not dissociate in water; that is, 100 molecules of sugar will go into solution forming, let us say, 100 active particles which take part in exerting osmotic pressure. Salts, acids, and bases, on the other hand, dissociate into ions. It is estimated, for example, that a thousandth normal ($n/1000$) solution of KCl is completely dissociated, as follows: $KCl = K + Cl$. One hundred molecules of KCl will therefore give 200 active particles in solution. What concentration of potassium chloride in water will be required to give the same osmotic pressure as that exerted by the sugar solution of problem 427?

CHAPTER V

SOUND

TRANSMISSION OF SOUND

78. Sound Waves.—Physically speaking, sound is that form of vibratory motion which may be perceived by the ear. All sound originates in vibrating bodies. Sound requires for its transmission a medium which is continuous, ponderable (weighable), and elastic. Sound is transmitted from point to point by means of waves. In those media which possess the property of rigidity the waves may be longitudinal or transverse; in media which do not possess rigidity (as air or water) sound is transmitted only by means of longitudinal waves.

Longitudinal waves consist of condensations (*c* and *c′*) and rarefactions (*r*), Fig. 61, *A*. *Transverse waves* are those in which the motion of the particles is at right angles to the direction of propogation, Fig. 61, *B*. *Wave length* is the distance measured from a given point in a wave to the corresponding point in the next wave of the same system, as from *c* to *c′*, Fig. 61, *A*, or from *a* to *b*, Fig. 61, *B*. *Amplitude* of vibration is measured by one-half the path swept out by the vibrating particle, as *ac* or *de*, Fig. 61, *B*.

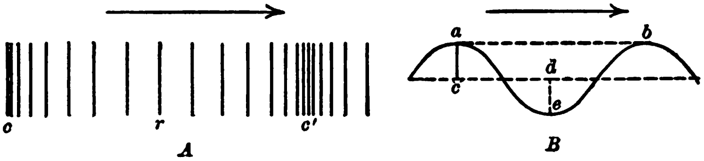


FIG. 61.—Longitudinal and transverse waves.

79. Intensity.—The intensity or loudness of sound varies (*a*) with the area of the sounding body; (*b*) the density of the medium; (*c*) directly as the square of the amplitude of vibration; and (*d*) inversely as the square of the distance from the source.

80. Velocity.—The velocity of sound may be computed by means of the equation

$$V = \sqrt{e/d},$$

in which V = velocity of sound; e = coefficient of elasticity of the medium, in absolute units, and d = density of the medium.

Example.—The coefficient of elasticity of copper is about 12×10^{11} dynes/cm^2; and its density 8.8 g/cm^3. Find the velocity of sound in copper, in feet per second.

Solution.—$V = \sqrt{e/d} = \sqrt{12 \times 10^{11}/8.8} = 369{,}300$ *cm/sec.* = 12,116 ft./*sec.*

Example.—The coefficient of elasticity of copper is 17.4×10^6 lb./in.^{2}; its specific gravity is 8.8. Find the velocity of sound in this metal in feet per second.

Solution.—Since in the equation $V = \sqrt{e/d}$, *e is expressed in absolute units, and since we wish to find the velocity in feet per second, it will be necessary to reduce* 17.4×10^6 *lb./sq. in. to absolute units (poundals/ft.*^{2}*). Now* 17.4×10^6 *lb./sq. in.* $= 17.4 \times 10^6 \times 32 \times 144 = 8{,}064 \times 10^6$ *poundals/sq. ft. Also, a specific gravity of* $8.8 = 62.5 \times 8.8 = 550$ *lb./cu. ft. Then* $V = \sqrt{e/d} = \sqrt{8{,}064 \times 10^6/550} = 12{,}100$ *ft./sec.*

81. Newton's Equation, and Laplace's Correction.—Newton derived the equation

$$V = \sqrt{p/d},$$

in which V = velocity of sound in air; p = pressure of the atmosphere; and d = density of the air. This equation, however, gave results which were only about 80 per cent. of the velocity of sound as determined by experiment. Laplace added the correcting factor 1.41 to take account of the coefficient of adiabatic expansion. The equation thus corrected, becomes

$$V = \sqrt{1.41p/d}.$$

82. Correction for Temperature.—The velocity of sound in air at 0°C is

$$V_o = 1{,}090\ ft./sec. = 332\ m/sec.$$

and for any temperature t

$$V_t = V_o\sqrt{1 + 0.003665 \times t}.$$

A change in temperature of 1°C causes a corresponding change in the velocity of sound in air (increase or decrease) = 2 *ft./degree* = 0.6 *m/degree.*

Problems

429. Find the velocity of sound in air in feet and meters when the temperature is +10°C; −10°C.

430. How far will sound travel in air in half a minute when the temperature is 68°F?

431. A given mass of air is contained in a rigid vessel of constant volume. By means of a force pump the mass of air in the tank is doubled. (*a*) How is p within the tank affected? (*b*) How is e affected? (*c*) The density d? (*d*) How is the velocity of sound affected?

432. The air in the tank (problem 431) is heated. How will this affect the velocity of sound, and why?

433. A column of air, under a pressure of 1 atmosphere is contained in a rigid cylindrical tank, into one end of which there is fitted a frictionless piston. If the air in the tank be heated; how will the velocity of sound through it be affected, when (*a*) the piston moves freely (*b*) when the piston is clamped rigidly in place?

434. If a pressure of 2 atmospheres be applied to the piston (problem 433) and the temperature of the air remain unchanged, how will the velocity of sound in the medium be affected?

435. Find the velocity of sound in air when the pressure is 72 cm, and the density is 0.00128 gram per cm^3.

436. A person approaching a large building at night stamps his foot on the pavement, and 0.8 sec. later hears the echo. How far is the person from the building, assuming the temperature to be 68°F?

437. Two large buildings are 346.4 ft. apart. The air is still and the temperature is 24°C. A pistol is fired at a distance of 118.8 ft. from one of the buildings and 227.6 ft. from the other. (*a*) How long before the man who fired the pistol will hear an echo? (*b*) When will he hear each of the next three?

438. The density of air is about 14.4 times that of hydrogen. Find the velocity of sound in hydrogen at 0°C, and at 20°C, the pressure in both cases being the same.

439. A cliff 460 ft. distant returns an echo to an observer in 0.8 sec. Find the velocity of the sound, and the temperature.

440. A given medium has a coefficient of elasticity of $1{,}225 \times 10^6$ grams per cm^2. The velocity of sound in this medium is 3,700 m per sec. Find the density of the medium.

441. If the velocity of sound in steel, density 7.8, be 5,000 m per sec., what is the coefficient of elasticity e in dynes per cm^2? How does this value compare with the value for Young's modulus for steel?

442. Consider the equation $V = \sqrt{e/d}$. Show that V and $\sqrt{e/d}$ have the same dimensions.

443. Calladon and Sturm found that the velocity of sound in water at 8.1°C is 1,435 m per sec. From this data find the coefficient of elasticity of water in (*a*) grams per square centimeter; (*b*) dynes per square centimeter.

444. On a given day when the temperature is -5°C, the barometer reads 74 cm. The density of the air is 0.001285. Find the velocity of sound, by two methods, and compare the two results obtained.

445. How does the coefficient of elasticity *e*, and the density *d* of water compare with that of air? How do you account for the fact that the velocity of sound in water is about five times as great as in air?

446. Determine the ratio of the velocity of sound in steel to that in brass. See Table V for Young's modulii.

RESONANCE, VELOCITY, WAVE LENGTH, AND PERIOD

83. Resonance.—If a tuning fork be held above a resonance tube, of such a length that the period of the vibrating air column is the same as that of the fork, the two (fork and tube) are said to be in resonance.

A *closed resonance* tube, Fig. 62, *A*, sounding its fundamental, represents one-fourth of the resulting wave length; an *open resonance tube*, Fig. 62, *B*, sounding its fundamental, represents one-half of the resulting wave length. That is,

closed resonance tube = ¼ *wave length;*
open resonance tube = ½ *wave length.*

84. Equation.—The relation between wave length, velocity, period, and frequency may be represented by

$$\lambda = VT = V/n$$

in which λ = wave length; V = velocity of sound in air at a given temperature; T = period of vibration; n = frequency (number of vibrations per second). It is important to bear in mind that the frequency is the reciprocal of the period; that is, $n = 1/T$.

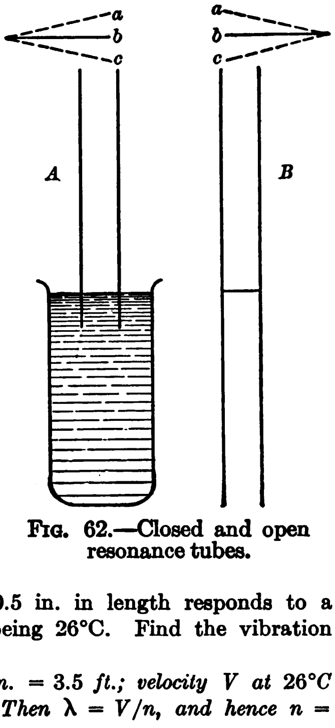


FIG. 62.—Closed and open resonance tubes.

Example.—A closed resonance tube 10.5 in. in length responds to a given fork, the temperature of the air being 26°C. Find the vibration rate (frequency) of the fork.

Solution.—Wave length $\lambda = 4 \times 10.5$ *in.* = 3.5 *ft.; velocity* V *at* 26°*C* = 1,090 + 26 × 2 = 1,142 *ft. per sec. Then* $\lambda = V/n$, *and hence* n = 1,142/3.5 = 326 *vibrations/sec.*

85. Reenforcement and Interference of Sound.—Two sound waves of the same phase, amplitude, and wave length reenforce each other at every point. Two sound waves of the same amplitude, wave length but of opposite phase may annul each other at every point. Two sound-wave systems, the waves of which are of different lengths, may alternately reenforce each other and interfere with each other, as shown in Fig. 63. The alternate rise and fall in the intensity of sound due to reenforcements and interference gives rise to *beats.*

The *number of beats* occurring per second is equal to the *difference in frequency* of the two sounding bodies.

86. Kundt's Experiment.—Kundt's apparatus, Fig. 64, furnishes a method for measuring the velocity of sound in metals. The metal rod *R* is clamped at the middle point. The length of this rod is one-half the wave length λ_m of the sound in the metal. The distance from one dust heap to the next in the tube is likewise one-half the wave length of the sound wave in air; that is, the distance between alternate dust heaps is λ_a. The relation of the velocities of the sound in the two media to the corresponding wave lengths is

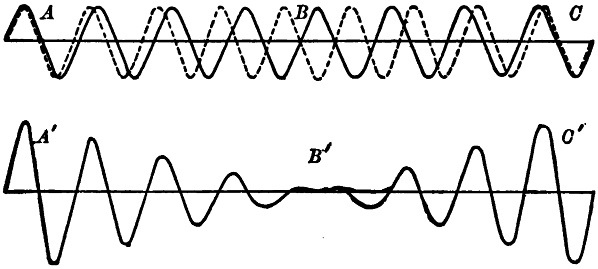


FIG. 63.—Interference of sound waves.

$$V_m : V_a = \lambda_m : \lambda_a$$

where V_m = velocity of sound in the metal; V_a = velocity of sound in air; λ_m = the wave length in the metal; λ_a = wave length in air.

Problems

447. A sounding body makes 100 vibrations per sec. Find the wave length of the disturbance in air at (*a*) + 20°C; (*b*) − 20°C.

448. A tuning fork gives off waves in air 130 cm in length, at 0°C. Find (*a*) the frequency of the fork; (*b*) its period.

449. A string makes 256 complete vibrations per sec., when the velocity of sound is 346 m per sec. Find the wave length.

450. A tuning fork makes 1,024 vibrations in a second, and the length of the sound wave given off is 32 cm. Find the velocity of the sound.

451. A cylindrical glass tube is placed vertically in water, and its length is adjusted until it responds to a fork making 256 vibrations per sec., when the temperature is 20°C. Find the length of the resonance tube in feet.

452. A closed resonance tube, 11.4 in. in length, responds to a fork making 300 vibrations per sec. Find the temperature.

453. Make drawings to illustrate wave length and amplitude of (*a*) transverse waves; (*b*) longitudinal waves.

454. Find the wave length in feet of sound in air due to 226 vibrations per sec., when the temperature is 68°F.

455. An open resonance tube responds to a fork making n vibrations per sec., when the temperature is t°C. Find (*a*) the length of the tube when n is 206, and t is 21.5; (*b*) the number of vibrations per second when t is 0°C.

456. In a certain experiment with a Kundt's apparatus the following data were obtained: Rod, 1 m in length and clamped in the middle. Average distance between dust piles, 10 cm. Temperature at time of experiment, 22°C. Find the velocity of sound in the rod.

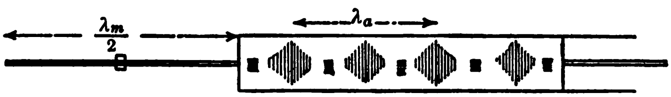


FIG. 64.—Kundt's apparatus.

457. Find the velocity of sound in brass from the following data. A brass rod used to excite the air in a Kundt's tube is 1 m long. The dust heaps which it produces in the tube are 99 mm apart. The temperature is 15°C.

458. If the density of the rod (problem 457) is 8 g/cm^3, what is its coefficient of elasticity e (Young's modulus)?

459. The velocity of sound in aluminum is 5,100 m per sec. Find its coefficient of elasticity.

460. A metal rod of length 125 cm, and density 7, is clamped at the middle. When struck on the end it gives a note of 1,200 vibrations per sec. Find its coefficient of elasticity.

461. Two wave trains have the same amplitude, and start from a given point in the same phase. The wave length of one is to that of the other as 6 : 10. Make a sketch of these two wave trains, and explain the significance of the resulting points of reinforcement and interference.

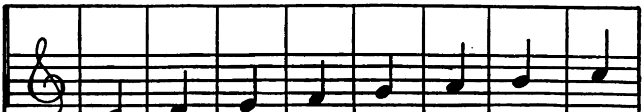
462. Two sounds are produced, making 100 and 120 vibrations per sec. respectively. Both sound wave trains travel with the same velocity. (*a*) When the first sound makes one vibration, the second sound makes how many? (*b*) What fraction of a wave length does the second gain over the first per vibration? (*c*) In what time will the second gain a whole wave length on the first? (*d*) How many times will this occur per second? (*e*) How many beats will occur per second?

PITCH

87. Diatonic Scales.—Pitch is that quality of sound which is determined by the number of vibrations per second. Pitch may be measured by means of a siren.

A diatonic scale consists of a series of eight notes having definite ratios. The *major diatonic scale* in the key of *C* is derived from the three major triads, having ratios 4 : 5 : 6, as follows:

$$\left.\begin{array}{l} C : E : G \\ G : B : D' \\ F : A : C' \end{array}\right\} = 4 : 5 : 6$$



Name	DO	RE	MI	FA	SOL	LA	TI	DO
Letter	*C*	*D*	*E*	*F*	*G*	*A*	*B*	*C'*
Ratio	1	9/8	5/4	4/3	3/2	5/3	15/8	2
Interval		9/8	10/9	16/15	9/8	10/9	9/8	16/15

FIG. 65.—Diatonic scale.

Starting with *C* as a keynote, and assigning to it a value of 256 vibrations per sec., and using the ratios of the major chord, we derive the major scale, Fig. 65, as follows:

	C	D	E	F	G	A	B	C,
Key of C..........	256	288	320	341	384	427	480	512

The *minor diatonic scale* is derived from the minor triads, the ratios being 10 : 12 : 15.

Transposition.—A scale having C as the keynote is sometimes called the natural scale. In order to accommodate different voices and instruments, it is frequently desirable to change the keynote of the scale from C to some other note, as for example, D, F, or G. In order to write the scale in any key we have only to select the vibration number corresponding to that letter, and to multiply this number successively by the fraction 9/8, 5/4, 4/3, etc.

Example.—Suppose that we wish to write the scale in the key of D. We select D from the diatonic scale as our keynote, its vibration number being 288. To get E we multiply 288 by 9/8 = 320; in a similar manner F = 288 × 5/4 = 360; G = 288 × 4/3 = 438, and so on.

88. Standards of Pitch.—In physics we assign to middle C 256 vibrations per sec. In music, however, the standard of pitch commonly employed is that which assigns to A 435 vibrations per sec.; this is called the *international standard* of pitch.

89. Tempered Scales.—In order to produce music in different keys on instruments having fixed keyboards, such as the piano and organ, it is necessary to determine upon some arbitrary ratio from note to note. In fixing the ratios from note to note on the piano, musicians have agreed to adopt a system known as that of *equal temperament;* that is, the ratio between all notes is equal. The ratio number selected is $\sqrt[12]{2} = 1.059$.

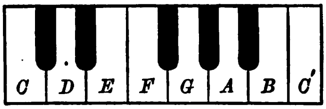


FIG. 66.—Thirteen keys comprising octave on piano.

On a piano there are 13 notes from C to C′, including eight white keys and five black keys. The black keys represent notes called sharps and flats. A sharp is a note having a vibration number higher than that of a given note; a flat is a note having a vibration number lower than that of a given note. Thus the first black key above C, Fig. 15, is the sharp of C and the flat of D. Taking the international pitch of A as 435 vibrations per sec., then C = 258.7; the sharp of C (first black key) = 258.7 × 1.059 = 274.1; D = 274.1 × 1.059 = 290.3, and so on.

A tempered scale such as that of the piano is sometimes called a *chromatic scale.*

90. Vibration of Strings.—A string may vibrate as a whole or in parts (segments). The fundamental tone is that given by a string vibrating as a whole; it is the tone of lowest pitch. Overtones are given off by a string vibrating in parts. Harmonics are overtones whose frequencies are exact multiples of the fundamental.

A node in a string is a point of minimum motion.

The relation of the frequency n to the length. stretching force, and density is represented by the equation

$$n = 1/(2L)\sqrt{F/m},$$

in which n = number of vibrations per second; L = length in centimeters; F = force in dynes; and m = mass of the string per unit length.

Example.—A string 1 m in length, and having a mass of 2 grams, is stretched by a force of 2 kg. Find the frequency of the string when it is sounding its fundamental.

Solution.—$L = 100$ *cm;* $F = 2{,}000 \times 980$ *dynes;* $m = 2/100$ *g/cm. Then* $n = 1/200\sqrt{196{,}000{,}000/2} = 49$ *vib./sec.*

91. Vibration of Air Columns.—The length of a *closed* pipe sounding its fundamental is one-fourth the wave length of the sound emitted; the length of an *open* pipe is one-half the wave length.

A node in a pipe is the point of *minimum motion* and *maximum change of density.*

A closed pipe is capable of producing only those overtones which correspond to odd multiples of the fundamental; that is, 3, 5, 7, etc. An open pipe is capable of producing all the overtones; that is, 2, 3, 4, 5, and so on.

Laws of Pitch.—(*a*) The pitch of a pipe varies inversely as its length. (*b*) The pitch of an open pipe is an octave higher than that of a closed pipe of the same length.

Problems

463. A siren is set so that its pitch is in unison with a given fork. The number of holes in the siren disc is 36, and it makes 90 revolutions every 10 sec. Find the frequency of the fork.

464. A current of air is blown against the disc of a siren having a row of 30 holes, while the disc is making 3,000 r.p.m. (*a*) What is the pitch of the resulting tone? (*b*) If the speed of the siren be doubled how will the pitch be affected?

465. Middle C is assumed to consist of 256 vibrations per sec. (*a*) Compute the vibration frequency of each of the other three notes of the major chord. (*b*) Compute the vibration frequency of each of the other three notes of the minor chord.

466. Beginning with 288 vibrations per sec., write a scale in the key of D, from C to C′ inclusive.

467. Beginning with 384 vibrations per sec. for G, write a scale in the key of G, from C to C′ inclusive.

468. If A on the piano has a frequency of 435 vibrations, find the frequency of (*a*) the next black key above A; (*b*) the first black key below.

469. A piano is tuned so that C makes 261 vibrations per sec. Using the ratio for equal temperament, find the frequency for each of the remaining 12 notes of the octave of the chromatic scale. How does the A note thus obtained compare with that of international concert pitch?

470. The tones of three forks form a major triad. The middle fork gives a note of 330 vibrations per sec. Find the vibration rate of the other two forks.

471. How will the pitch of a string be affected (*a*) if its length be doubled? (*b*) if its tension be quadrupled? (*c*) if the mass per unit length be increased ninefold?

472. A given string stretched by a force of 1 lb. makes 200 vibrations per sec. (*a*) How will the frequency be affected if the stretching force is increased to 4 lb.? (*b*) How will the pitch be affected?

473. An aluminum wire 1 m in length, and 1 mm in diameter, is stretched by a force of 4 kg. Find the pitch of its fundamental.

474. A string 100 cm long produces middle C. A bridge is placed under the string, and its position adjusted until the string produces E of the same octave. (*a*) Where is the bridge placed? (*b*) Where should it be placed to produce the other two notes of the major chord?

475. If the stretching force upon a certain string is 500 grams, and it sounds middle C, what will be the force necessary to tune it (*a*) to D of the natural scale? (*b*) to E of the piano scale?

476. A wire 3 ft. long and stretched with a force of 9 lb. makes 300 vibrations per sec. Another wire of the same material and size is 4 ft. long, and is stretched with a force of 16 lb. Find the vibration frequency of the second wire.

477. A given wire 1 m in length, and stretched by a weight of 400 grams, makes 100 vibrations per sec. If the length be doubled, and the stretching force be increased to 1,600 grams, how will the pitch be affected?

478. What is the relation of the length of a pipe to its pitch? What is the relation of the pitch of an open pipe to that of a closed pipe? An open pipe of given length is sounding its fundamental. Suppose that a person stop one end by means of a card. How will the pitch be affected?

479. Define node in a pipe, and explain wherein it differs from a node in a string.

480. Determine the length of an open organ pipe that is in unison with E above middle C of the piano when the temperature of the air is 25°C. End corrections are not to be considered in this problem.

481. What is the vibration frequency of an open organ pipe 32 ft. long when the temperature is 24°C? What effect would be produced upon the ear by the waves from a closed pipe of the same length?

482. A long glass tube 5 cm in diameter is so arranged that water may be forced in at the bottom, thus varying the length of the air column above the water. With an E fork (320 vibrations per sec.), strong resonance occurs when the air column is 25.1 cm long and again when it is 75.3 cm long. Temperature of the air is 25°C. Determine the velocity of sound at 0°C. Why is not one of the observed lengths exactly three times the other?

483. Two wires of the same material and the same size have the same pitch. One is stretched with a force of 4,900 grams and is 80 cm long; the other is stretched with a force of 8,100 grams; how long is the second wire?

484. What is the mass per unit length of the wire of the preceding problem, if the vibration frequency of the one 80 cm long is 128 per sec.?

485. A whistle makes 500 vibrations per sec. when the temperature is 15°C. How many sound waves will occur between the whistle and an observer 1,120 ft. distant? How many vibrations fall upon his ear (*a*) when he is standing still? (*b*) when moving toward the whistle at the rate of 40 ft. per sec.? (*c*) when moving away from the whistle at the rate of 40 ft. per sec.?

486. The frequency of the lowest continuous sound which the ear can perceive as a definite note is about 20 per sec; the limits for the highest, is about 40,000 per sec. Find the corresponding wave lengths in centimeters at 0°C.

487. The vibration frequency of a locomotive whistle is 760 per sec. The velocity of the train is 60 ft. per sec.; the temperature of the air 25°C. What is the vibration frequency of the sound heard by an observer (*a*) on the train; (*b*) on the track ahead of the train; (*c*) on the track behind the train?

488. The whistle on a train has a frequency of 500 vibrations per sec. The train is moving with a velocity of 30 miles per hr. The temperature of the air is 15°C. Find the change in pitch as the train passes an observer.

489. A locomotive approaches a man standing near the track. The vibration frequency of the bell seems to him to be that of high C (512 vibrations per sec.). After the train has passed him, the pitch seems to be that of A of the octave below (426.6 vibrations per sec.). The speed of the train is constant and the temperature is 0°C. Find the speed of the train.

490. Make a drawing of Lissajous' figures for periods in (*a*) the ratio of 2 to 1 (that is two spaces on the x-axis and one space on the y-axis); (*b*) ratio of 3 to 2.

491. Design a Blackburn's pendulum that will trace a figure 8.

CHAPTER VI

HEAT

TEMPERATURE MEASUREMENTS

92. Heat and Temperature.—*Heat* is a form of energy which is capable of affecting the physical condition of a body and which is accompanied in general by a change of state or temperature. Heat may be measured in thermal units (calories or B.t.u.), or in units of work (ergs, joules, gram-centimeters, foot-pounds).

Temperature is the condition of a body which affects our sensations of warmth and cold. Temperature changes are accompanied by certain physical changes, as for example changes in pressure, volume, electrical resistance, electromotive force. Any one of these changes may be chosen as a basis for temperature measurements. We shall consider, first, temperature measurements based on volume changes, as illustrated by the liquid-in-glass thermometer.

93. Thermometric Substances.—*Mercury.*—The limitations of a thermometric substance are in general fixed by its freezing point and its boiling point. Mercury freezes at − 38.8°C; boils at + 357°C under a pressure of 1 atmoshere. It is possible to increase the boiling point of a mercury-in-glass thermometer to 500°C, by increasing the pressure of the gas enclosed within the instrument.

Alcohol.—For measuring temperatures below −38.8°C alcohol is often used as the thermometric substance. This liquid is usually colored red or blue to render it visible against the glass. The freezing point of alcohol is − 130°C; its boiling point + 78°C.

Toluene.—The freezing point of toluene is −80°C; its boiling point + 110°C.

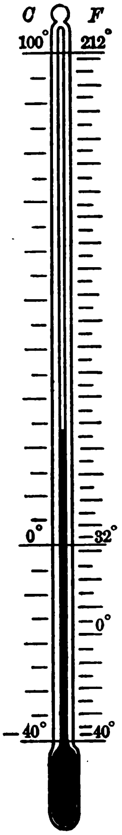


FIG. 67.—Centigrade and Fahrenheit scales.

94. Thermometric Scales.—On the *Centigrade scale* the interval between the freezing point and the boiling point is divided into 100 grades or degrees. On the *Fahrenheit scale* the interval between the freezing point and the boiling point is divided into 180°. The zero of this scale is 32° below the freezing point, thus making the interval from the zero of the scale to the boiling point 212°. The relation of the two scales is shown in Fig. 67. Since 100°C is equivalent to 180°F, we may write,

1C *unit* = $\frac{9}{5}$ F *unit*, and
1F *unit* = $\frac{5}{9}$C *unit*. Then
$(C \times \frac{9}{5}) + 32 = F$, and
$(F - 32)\frac{5}{9} = C$

Example.—A reading of −20°C is equivalent to what reading on the F scale? *Solution.*—$(-20 \times \frac{9}{5}) + 32 = -4$°F.

Example.—A reading of + 23°F is equivalent to what reading on the C scale? *Solution.*—$(23 - 32)\frac{5}{9} = -5$°C.

Example.—A reading of − 13°F is equivalent to what reading on the C scale? *Solution.*—$(-13 - 32)\frac{5}{9} = -25$°C.

Problems

491. Give the following C readings their equivalents on the F scale (*a*) +10°; (*b*) −10°; (*c*) +30°; (*d*) −30°; (*e*) −40°.

492. Give the following F readings their equivalents on the C scale? (*a*) +32; (*b*) +77; (*c*) +5; (*d*) −4°; (*e*) −40°.

493. (*a*) Change the following F readings to C: +95°, +14°, −49°. (*b*) Change the following C readings to F: +230°, −15°, −273°.

494. What is the equivalent on the C scale of the following F readings: (*a*) 18° above freezing; (*b*) 18° below freezing; (*c*) 18° above zero; (*d*) 18° below zero.

495. Change the following C readings to F: (*a*) 20° above freezing; (*b*) 20° below freezing; (*c*) −15°; (*d*) 0°.

496. Find the temperature on the C thermometer when a F thermometer reads (*a*) +72°; (*b*) −14°; (*c*) +1,000°.

497. Reduce the following readings on the C scale to corresponding readings on the F scale: (*a*) +25°; (*b*) −20°; (*c*) + 700°.

498. A thermometer tube of uniform bore has 18 F divisions to the centimeter. How many divisions per centimeter would there be if it were graduated to give C readings?

499. At what temperature is the C reading twice the F reading?

500. At what temperature is the F reading twice the C reading?

501. At what temperature does the F thermometer read (*a*) the same as the C thermometer? (*b*) twice as much? (*c*) three times as much?

502. It is found on test that the freezing-point mark of a given thermometer is −0.2°, and the boiling point +100.8°. What is the true temperature when this thermometer registers 50°, assuming the tube to be uniform in bore?

503. In testing a C thermometer it was found that the scale readings for the boiling point was +101°, and for the freezing point +0.5. What is the true reading when the mercury stands at the 60° mark on this scale?

EXPANSION

95. Coefficients of Expansion.—*Coefficient of linear expansion* is increase in length, per unit length, per degree. That is, $\alpha = l/Lt$ in which α is the coefficient of linear expansion; l is the change in length; L is the original length of the body; and t is the change in temperature. The relation between the length at 0°C (L_o) and the length at t degrees is

$$L_t = L_o(1 + \alpha t)$$

The *coefficient of volume expansion* (β) is the increase in volume, per unit volume, per degree. We may write

$$V_t = V_o(1 + \beta t)$$

It may be shown that, as a first approximation, $\beta = 3\alpha$.

NOTES.—(*a*) In cubical, as in linear expansion, there exists no strict proportionality between increase of volume and increase of temperature. This explains why the reading of thermometers filled with different liquids, such as mercury and alcohol, do not exactly agree.

(*b*) If an empty flask be heated, it expands as if it were solid throughout. See problem 510.

(*c*) The coefficient of volume expansion β increases considerably with increase of temperature, and becomes quite large near the boiling point. The equation $V_t = V_o(1 + \beta t)$ should therefore be considered only an approximation, and in the case of liquids, when temperatures are taken near the boiling point, it is better to write $V_t = V_o(1 + \beta t + \beta' t^2)$. In connection with this equation, note problems 511 and 512. See also Table XI.

Problems

504. A steel wire 7.7 ft. in length at 0°C, elongates 0.1 in. when heated to 90°C. Find the coefficient of linear expansion of brass. See Table IX.

505. A given steel rod (nickel-steel) increases 1 in. when heated from 0 to 200°C. Find the length of the rod at 0°C.

506. If steel rails, 30 ft. in length, are laid at a temperature of 59°F, how large a gap in inches, must be left between the ends, if the highest temperature allowed for be 113°F, assuming the coefficient of linear expansion of steel to be 0.0000115 per degree Centigrade.

507. A certain distance measured with a steel tape was found to be 1,025 ft. The measurement was made when the temperature was 30°C. The tape was correct at 16°C. What was the actual distance measured, the coefficient of expansion of the tape being 0.000012?

508. A steel bridge of 200 ft. span will change in length by how many inches when the temperature rises from −20°C to +20°C, the coefficient of linear expansion being 0.000012?

509. Two similar wires A and B, at 0°C, length of each 10 m, cross-sectional area 0.01 cm^2, Young's modulus 20×10^8 grams per cm^2, coefficient of expansion 12×10^{-6} are caused to elongate, one by a stretching force, and the other by heating. The wire A is stretched by a weight of 1 kg. To what temperature must B be raised to have an equal length?

510. A flask made of glass having a linear coefficient of expansion of 0.000008 is calibrated to hold 1,000 cc at 0°C. How much will it hold at 100°C?

511. At 0°C the volume of a given mass of alcohol is 1,000 cc. (*a*) What will be its volume at 10°C? (*b*) 70°C? See note (*c*), Art. 95.

512. A given mass of ether has a volume of 250 cc at 0°C. Find its volume at 5° below its boiling point.

513. A certain mass of mercury has a volume of 120 cc at 0°C, and a volume of 121.32 cc at 60°C. Find the coefficient of volume expansion of mercury.

514. A solid displaces 500 cc when immersed in water at 0°C, and displaces 503 cc when immersed in water at 30°C. Find (*a*) the mean coefficient of cubical expansion of the solid; (*b*) its coefficient of linear expansion.

515. Find the change in density of iron when heated from 0° to 200°C.

CHANGE OF VOLUME AND PRESSURE IN GASES

96. Absolute Zero.—If the volume of a given mass of gas be kept constant, we may write

$$P_t = P_o(1 + \alpha t),$$

in which P_t = pressure exerted by the gas at a given temperature; P_o = pressure at 0°C; and α = a coefficient which is called the *coefficient of pressure* for constant volume, and which is numerically equal to the coefficient of expansion for constant pressure. This means that $\alpha_v = \alpha_p = 1/273 = 0.00366$. If now we select a temperature t such that the pressure is zero, then $P_t = 0$, and from the equation $P_t = P_o(1 + \alpha t)$ we have $t = -273°\text{C}$; that is

$$\textit{Absolute Zero} = -273°\text{C}.$$

97. To Change Centigrade and Fahrenheit Readings to Absolute.—Since absolute zero = $T = -273°\text{C}$, and since $F = (C \times 9/5) + 32$, we may write

$$\textit{Absolute T on C scale} = C + 273, \text{ and}$$
$$\textit{Absolute T on F scale} = F + 459.4$$

Example 1.—Find the reading on the absolute scale of (*a*) + 20°C; (*b*) − 20°C.

Solution.—(*a*) + 20 + 273 = 293 *Abs. C;* (*b*) − 20 + 273 = 253 *Abs. C.*

Example 2.—Freezing point of the F scale is what reading on the absolute scale?

Solution.—*Freezing point* F = + 32. *Then* 459.4 + 32 = 491.4 *Abs. F.*

98. Boyle's Law.—According to Boyle's law, the product of pressure and volume of a gas is a constant, the temperature being constant. In other words, for conditions of constant temperature

$$pv = c = p'v'$$

where p and p' = the pressures exerted upon the gas; v and v' = the corresponding volumes; and c = a constant. For example, if the volume of a given mass of gas, under a pressure equivalent to 76 cm, be 100 cm³, then $pv = c = 7{,}600$. If now the pressure be reduced to 38 cm, the volume will become 200, and as before $p'v' = c = 7{,}600$.

It should be noted that the equation $pv = c$, for constant temperature is the equation for a rectangular hyperbola, and the curve, Fig. 68, represents an *isothermal line*, the characteristics of which are that $pv = p'v' =$ *constant*, which means that the area pv, Fig. 68, is equal to the area $p'v'$.

Example.—If for constant temperature conditions a given mass of gas, under a pressure of 80 cm, is 300 cm³, what will be the volume under a pressure of 60 cm?

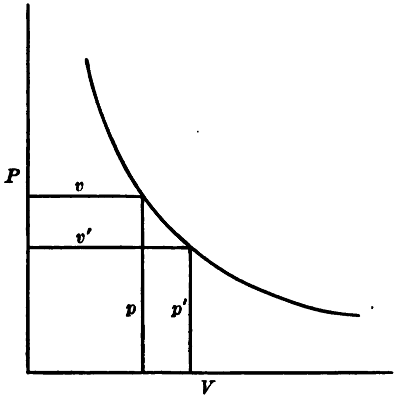


FIG. 68.—Isothermal curve.

Solution.—*Since* $pv = p'v'$, 80 × 300 = 60 × v', *and therefore* v = 400 *cm³.*

99. Gay-Lussac's Law.—*Gay-Lussac's law* (law of Charles) states that, "For a perfect gas the product of pressure and volume is proportional to the absolute temperature." In equational form

$$pv/p'v' = T/T'.$$

Example.—Under a pressure equivalent to 1 atmosphere (76 cm of mercury) and a temperature of −3°C the volume of a given mass of gas is 1 liter. Find its volume when the pressure is 72 cm and the temperature is 37°.

Solution.—$T = 270$ *and* $T' = 310$. *Then, since* $pv/p'v' = T/T'$ *we have* $(76 \times 1{,}000)/(72 \times v') = 270/310$, *from which* $v' = 1{,}211.9$ *cc.*

100. The General Gas Law.—Combining Boyle's law with that of Gay-Lussac (law of Charles) and using absolute temperature, we may write

$$pv = mRT,$$

in which p = pressure exerted upon the given gas; v = corresponding volume; m = mass of the gas; T = absolute temperature; and R = the gas constant. This constant may be determined by assigning known values to p, v, m, and T.

R in C.G.S. Units, Centigrade.—We select for the value of m 1 gram molecule of the gas chosen, because for a pressure of 1 atmosphere and a temperature equal to the freezing point, the volume of 1 gram molecule of any gas is known; that is v = 22,400 cc. Then from the equation $pv = RT$, we may write $76 \times 13.6 \times 980 \times 22{,}400 = R \times 273$. Hence R = 83,000,000 ergs/degree C.

R in English Units, Fahrenheit.—We wish to find the value of R in terms of English units, using air as the gas under consideration. In this case we select m as the mass of 1 lb. of air under standard conditions because the volume for these conditions is known; that is, v = 12.39 cu. ft. The value for T = number of degrees *from absolute zero to the freezing point*, in the F scale = 491.4. Then from the equation $pv = mRT$, in which $m = 1$, we write $R = 14.7 \times 144 \times 12.39/491.4$ = 53.37 ft.-lb./degree F.

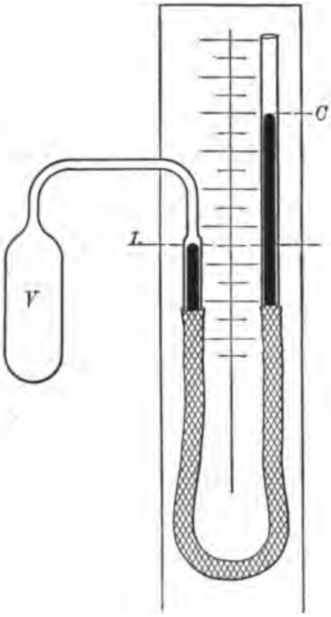


Fig. 69.—Air thermometer.

It must be noted that in finding the value of R we have considered air as the gas under pressure. If we wish to determine R for some other substance, such as superheated steam for example, we shall have to find the value for unit mass under standard conditions as before. In the case of superheated steam R = 85.5 ft.-lb. per degree F.

Example.—The molecular weight of oxygen is 32; a gram molecule of oxygen has a mass of 32 grams. (*a*) How many gram molecules of this gas enclosed in a vessel having a volume of 50 liters will exert a pressure of 1,245,000 dynes per cm^2 at 27°C? (*b*) How many grams?

Solution.—(*a*) *From the equation* $pv = mRT$ *we may write* $m = pv/RT = 1{,}245{,}000 \times 50{,}000/83{,}000{,}000 \times 300 = 2.5$ *gram molecules.* (*b*) *The mass of* 1 *gram molecule of oxygen is* 32 *grams. Then the total mass* $= 2.5 \times 32 = 80$ *grams.*

Example.—A tank contains 10 lb. of air at a temperature of −9.4°F under a pressure of 200 lb. per sq. in. Find the volume of the air.

Solution.—$v = mRT/p = (10 \times 53.37 \times 450)/(200 \times 144) = 8.34$ *cu. ft.*

101. The Constant Volume Gas Thermometer.—Fig. 69 shows in outline the essential parts of a standard hydrogen thermometer. The volume of the gas in V is kept constant by raising or lowering the open tube C, thus keeping the mercury at L at a constant level. With this instrument the temperature is measured in terms of the pressure exerted. Starting with the equation $P = P_0(1 + \alpha\theta)$ we may write

$$\theta = 100(P - P_0)/(P_{100} - P_0),$$

where θ = temperature in degrees on the presssure-temperature scale; P = pressure exerted by the gas in V at the temperature θ; P_0 = pressure corresponding to the zero of the scale; that is, when V is placed in melting ice; P_{100} = pressure corresponding to the boiling point of water, under 1 atmosphere's pressure. When P_0 and P_{100} are once fixed for a given instrument, the temperature may be determined by finding the value of P.

Example.—When the bulb of an air thermometer was placed in ice, the mercury column stood at a point 20 mm above the line L, Fig. 69; when placed in boiling water, the pressure was equivalent to 305.6 mm. When the bulb was placed in a given bath, the pressure was 105.6 mm. The barometric reading was 76 cm. throughout the entire experiment. Find the temperature θ of the bath.

Solution.—Since the pressure of the atmosphere during the experiment = 760 *mm,* $P_0 = 20 + 760 = 780$ *mm;* $P_{100} = 305.6 + 760 = 1065.6$ *mm; and* $P = 865.6$ *mm. Then* $\theta = 100(865.6 - 780)/(1065.6 - 780) = 30°$.

Problems

516. Change the following C readings to Absolute Centigrade: (*a*) +10°; (*b*) −10°; (*c*) freezing point; (*d*) boiling point.

517. (*a*) How many C degrees are there from freezing point to Absolute zero? (*b*) How many F degrees from freezing point to Absolute zero? (*c*) How many F degrees from zero F to Absolute zero?

518. Change 491.4 F degrees Absolute to (*a*) Fahrenheit; (*b*) Centigrade.

519. Change 373 C degrees Absolute to (*a*) Centigrade; (*b*) Fahrenheit.

520. Change the following F readings to Absolute on the F scale: (*a*) +68; (*b*) −10; (*c*) freezing point; (*d*) boiling point.

521. Change the following Absolute C readings to Centigrade: (*a*) 373; (*b*) 293; (*c*) 233. (*d*) Give the equivalent F readings.

522. Change the following Absolute F readings to Fahrenheit: (*a*) 491.4; (*b*) 459.4; (*c*) 671.4. (*d*) Give the equivalent C readings.

523. A given mass of gas at a temperature of 0°C, under a pressure of 76°C occupies a volume of 1 liter. Find the volume when the pressure is 72 cm and the temperature 100°C.

524. A mass of gas has a volume of 100 cc at a temperature of −3°C and pressure of 74 cm. Find the pressure if the volume be kept constant and the temperature be changed to +27°C.

525. When the barometric pressure is 30 in. and the temperature is 32°F, the volume of a given mass of gas is 10 cu. ft. Find the volume when the barometric pressure is 28 in. and the temperature (*a*) −10°F; (*b*) 10°F above freezing point.

526. The capacity of a metal tank is 8 cu. ft. The tank is filled with air under standard conditions (P = 14.7 lb./sq. in. and t = 32°F). Find the pressure in pounds per square inch when the temperature is increased to 212°F.

527. A liter of gas at 100°C will have what volume at 300°C, of the pressure remains constant?

528. A liter of gas at 100°C and 76 cm pressure, will exert what pressure if the temperature be raised to 200°C without changing the volume.

529. A liter of air at 23°C and 50 cm pressure will have what volume under standard conditions?

530. If the density of air is 1.293 grams per liter under standard conditions, what density will it have at 80°C and 60 cm pressure?

531. The value of R in the gas equation ($pv = RT$) is 83,000,000 ergs per degree Centigrade. Find the value of R in calories per degree.

532. Find the volume of 5 gram molecules of a gas at a temperature of 270°C. and under a pressure of 2 atmospheres ($pv = mRT$).

533. The molecular weight of oxygen is 32. If 64 grams of oxygen be enclosed in a vessel having a volume of 1,000 cc at a temperature of 27°C, what pressure will it exert in (*a*) dynes; (*b*) grams?

534. If 10 lb. of air in a tank at 68°F exert a pressure of 100 lb. per sq. in., what is the volume of the tank?

535. If 10 lb. of air under a pressure of 100 lb. per sq. in. occupy a volume of 20 cu. ft. what is the temperature?

536. A vessel containing nitrogen (molecular weight 28) has a volume of 50 liters. When the temperature is 27°C the pressure is 70 cm. Find the number of grams of N in the vessel.

537. A volume of 50 cc of hydrogen is collected in a tube over mercury. The mercury in the tube stands 20 cm above that in the bath. The barometer reads 74 cm and the temperature is 27°C. Hydrogen weighs 0.0896 gram per liter under standard conditions. Find the weight of hydrogen in the tube.

538. A liter of air under standard conditions of pressure and temperature has a mass of 1.293 grams. What mass of air will a liter flask contain at −50°C, and a pressure equivalent to 160 cm of mercury?

539. At what temperature will a liter of gas under 1 atmosphere's pressure and at 40°C become 1.2 liters, if during the change in temperature the pressure becomes one-fourth?

540. Ten pounds of air under a pressure of 200 lb. per sq. in. has a volume of 10 cu. ft. Find the temperature F.

541. A tank contains 10 lb. of air, temperature 70°F, pressure 200 lb. per sq. in. Find the volume of the air.

542. The volume of a given mass of hydrogen is 300 cc when the temperature is 27°C and the pressure equivalent to 750 mm of mercury. Find the volume of the gas when the temperature is 50°C and the pressure 800 mm of mercury.

543. A hydrogen thermometer is used to measure the temperature of the water in a certain tank. Let L, Fig. 69, represent the level of the mercury in the closed tube. Let C be the level of the mercury in the open branch. The volume of air in the bulb V is kept constant. When the bulb V is in melting ice C is 10 cm above L. When V is in steam above boiling water at a pressure of 1 atmosphere, C is 41.5 cm above L. When V is in the water whose temperature is to be found, C is 18.5 cm above L. Find the temperature of the water, the barometric pressure being 76 cm. throughout the experiment.

544. Consider the hydrogen thermometer of problem 543. What will be the height of the mercury column above L when the bulb V is placed in a vessel containing liquid the temperature of which is (*a*) +30°; (*b*) −30°?

HEAT MEASUREMENTS

102. Heat Units.—The idea of *quantity of heat* involves three factors namely, *mass, specific heat,* and *change of temperature.* Quantity of heat may be measured in terms of calories, or in terms of British thermal units (B.t.u.).

A *calorie* is the heat required to raise the temperature of 1 gram of water 1°C. When very accurate determinations are required, the 1° is understood to mean from 15° to 16°C.

A *B.t.u.* is the heat required to raise 1 lb. of water 1°F.

103. Specific Heat and Thermal Capacity.—The *specific heat* of a substance is the heat (calories or B.t.u.) required to raise unit mass (gram or pound) 1° (Centigrade or Fahrenheit). The sp. h. of water = 1. The sp. h. of copper, for example = 0.09; this means that it requires 0.09 cal. to raise 1 gram of copper 1°C, or 0.09 B.t.u. to raise 1 lb. of copper 1°F.

The *thermal capacity* of a body is the heat (calories or B.t.u.) required to raise its temperature 1°. The thermal capacity is therefore a quantity which is equal to the product of the mass of the body multiplied by its specific heat, or

$$\textit{Thermal capacity} = m \times s,$$

where m = mass in grams or pounds, and s = specific heat.

Example.—A copper vessel has a mass of 1,050 grams, which is equivalent to 2.315 lb. The sp. h. of copper is 0.09. Find the thermal capacity of this vessel in (*a*) calories per degree C; (*b*) B.t.u. per degree F.

Solution.—T.C. $= m \times s =$ (*a*) $1{,}050 \times 0.09 = 94.5$ *cal./degree C;* (*b*) $2.315 \times 0.09 = 0.20835$ *B.t.u./degree F.*

104. Specific Heat by Method of Mixtures.—Specific heat is usually determined by the so-called method of mixtures. A hot body is dropped into water (or other liquid) contained in a calorimeter. The temperature of the hot body A falls; the temperature of the water B and the calorimeter C rises. We assume that the *heat lost by* A = *heat gained by* $B + C$; that is

$$m(t_1 - t_2)s = m'(t_2 - t_3)s' + m''(t_2 - t_3)s''$$

where m = mass of the body; $(t_1 - t_2)$ = its change of temperature; s = sp. h. of body; m' = mass of water (or other liquid) in calorimeter; $(t_2 - t_3)$ = its change of temperature (rise); s' = sp. h. of liquid in calorimeter; m'' = mass of calorimeter; $(t_2 - t_3)$ = change of temperature; and s'' = sp. h. of calorimeter.

Example.—Five hundred grams of lead shot at a temperature of 98°C are poured into 350 grams of water contained in an iron vessel having a mass of 300 grams. The initial temperature of the water in the calorimeter was 20°C; its temperature after the addition of the lead shot is 23°C. Find the sp. h. of the lead.

Solution.—From the Specific Heat Tables, page 197, *we find that the sp. h. of iron* $s'' = 0.116$. *Then* $500 \times (98 - 23) \times s = 350 \times (23 - 20) \times 1 + 300(23 - 20) \times 0.116$, *from which* $s = 0.03$.

105. Heats of Fusion and Vaporization.—The *heat of fusion* of a substance is the heat (calories or B.t.u.) required to change unit mass from a solid to a liquid, without change of temperature.

The *heat of fusion of ice* at the melting point and for 1 atmosphere's pressure = 80 *cal./gram* = 144 *B.t.u./lb.*

The *heat of vaporization* of a substance is the heat (calories or B.t.u.) required to change unit mass of the substance at a given temperature from a liquid to a vapor without change of temperature.

The *heat of vaporization of water* at the boiling point for 1 atmosphere's pressure = 538 *cal./gram* = 970 *B.t.u./lb.*

106. Heat of Combustion.—The *heat of combustion* is the heat (calories or B.t.u.) liberated when unit mass of the substance is burned. For example the H.C. of anthracite coal is about 8,000 cal./gram = 14,000 B.t.u./lb.

Heat of Combustion Tables, see page 198.

107. Mechanical Equivalent of Heat.—The *mechanical equivalent of heat* (Joule's equivalent), usually designated by the letter J, expresses the relation between heat and mechanical work as follows:

$$1\ cal. = 4.186 \times 10^7\ ergs,$$
$$1\ B.t.u. = 778\ ft.\text{-}lbs.$$

Example.—How much energy is expended in changing 20 grams of ice at − 5°C to steam at 120°C, the sp. h. of ice being 0.5, and the sp. h. of steam for the given condition being 0.46. Express the result in (*a*) calories; (*b*) ergs.

Solution.—The heat required to bring about the various changes from ice at −5°, *to steam at* 120° *may be summarized as follows:*

Change from −5 *to* 0 = 20 × 5 × 0.5 = 50 *cal.*
Change from ice to water at 0° = 20 × 80 = 1,600 *cal.*
Change from 0° *to B. P.* = 20 × 100 = 2,000 *cal.*
Change from water to steam at 100° = 20 × 5 38 = 10,760 *cal.*
Change from 100° *to* 120° = 20 × 20 × 0.46 = 184 *cal.*

Total heat required = 14,594 *cal.*

This is equivalent to 14,594 × 4.186 × 10^7 *ergs* = 61,090 *joules.*

108 Carnot's Cycle. Efficiency.—This cycle, Fig. 70, represents an ideal cycle for a perfect engine using a perfect gas. The curves AB and CD represent *isothermal changes* in pressure and volume; curves BC and DA represent *adiabatic changes.*

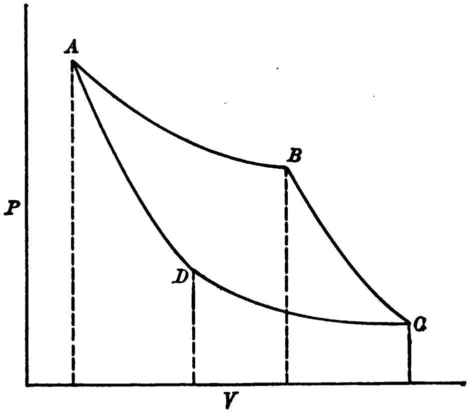


Fig. 70.—Carnot's cycle.

The equation for an isothermal curve, representing Boyle's law, is $pv = c =$ *constant;* the equation for an adiabatic curve is $pv^\gamma = c$, where $\gamma = 1.41$.

During the isothermal expansion AB, H units of heat are absorbed; during the isothermal compression, H' units of heat are given out. During the whole cycle, external work is done equal to $W = ABCD$. This work has been obtained by the transformation of $H - H'$ units of heat; that is, $W = H - H'$.

The *efficiency of the cycle* is the ratio of the useful work W gotten out of the engine, to the total amount of energy H put in. Since it may be shown that $(H - H')/H = (T - T')/T$, we may write

$$\textit{Efficiency} = W/H = (H - H')/H = (T - T')/T$$

where W = useful work out; H = heat energy put in; H' = heat out; T = Abs. T. of gas taken in; and T' = Abs. T. of gas ejected.

Example.—A perfect engine is represented as taking steam from a boiler at 127°C, and exhausting it at 27°C. Find the efficiency.

Solution.—*Efficiency* $= (T - T')/T = (400 - 300)/400 = 25$ per cent.

109. Conduction.—Heat is transmitted through a body by conduction. The rate at which conduction occurs is represented by

$$H = kA\tau(t - t')/l$$

When c.g.s. units are employed, H = calories; k = coefficient of conductivity ("conducting power") of the given material; A = area considered, in cm²; $(t - t')$ = difference in temperature (Centigrade) between the surfaces through which conduction takes place; τ = time in seconds; l = thickness of the conducting material.

When English units are used, H = B.t.u.; k = conducting power of the medium; A = area in *square feet;* $(t - t')$ = difference in temperature Fahrenheit; τ = time in hours; l = thickness in *inches.*

Table of Coefficients of Thermal Conductivity, page 199.

Example.—How much heat will be conducted through a plate glass window, 3 by 4 ft., ¼ in. in thickness, from 7 a.m. to 10 p.m., the temperature inside the room being 20°C, and outside 5°C?

Solution.—The value of k for glass (Table XXII) *is* 7; *the difference of temperature* = 27°*F; and the time* = 15 *hr. The heat transmitted through the glass, then, is* $H = (7 \times 3 \times 4 \times 27 \times 15)/0.25 = 136{,}080$ *B.t.u.*

Problems

545. How much heat in calories is required to change 10 grams of ice at −10°C to steam at 100°C, the pressure being 76 cm? *Ans.* 7,280 cal.

546. How many B.t.u. will be required to change 10 lb. of ice at 0°F to steam at 212°F? *Ans.* 13,100 B.t.u.

547. A piece of lead, mass 340 grams, is heated to 90°C and is then dropped into 300 grams of water contained in an aluminum calorimeter. The initial temperature of the water and the calorimeter is 24°C, the final temperature 26°C. Find the mass of the calorimeter. *Ans.* 120 grams.

548. Ten pounds of steam under a pressure of 1 atmosphere and at 212°F are mixed with 100 lb. of water at 72°F and 3 lb. of ice at 22°F. Find the resulting temperature of the mixture. *Ans.* 113 lb. of water at 169°F.

549. Find the energy in joules necessary to warm 50 kg of copper through a temperature range of 60°C.

Ans. 113,022 joules.

550. Find the energy in foot-pounds necessary to melt 100 lb. of ice and warm the resulting water to 88°F.

Ans. 15,560,000 ft.-lb.

551. What horsepower could change ice at 32°F to water at 212°F at the rate of 5 lb. per min.? *Ans.* 38 hp.

552. A train of mass 300 tons has a speed of 60 ft. per sec. How much heat in B.t.u. is developed at the brakes when it is stopped? *Ans.* 43,380.4 B.t.u.

553. A liter of water is heated from 20°C to the B. P. (100°C). Find (*a*) the number of calories consumed; (*b*) the energy required in ergs.

554. If we heat a quart of water (2 lb.) from 52°F to the B. P., (*a*) how many B.t.u. are consumed? (*b*) How many ft.-lb. of energy are put into the water?

555. How many calories of heat will be required to change 20 grams of ice at −10°C, (*a*) to water at 60°C? (*b*) steam at 100°? (*c*) steam at 120°?

556. A piece of copper (sp. h. = 0.09) having a mass of 1 kg cools from 96°C to 0°C. (*a*) How much heat is liberated by the copper? (*b*) How many grams of ice will this heat melt?

557. How many calories of heat will be required to change 100 grams of water at 20°C to steam at 150°C, the specific heat of steam for the given temperature and pressure being 0.46?

558. How many B.t.u. are required to change 20 lb. of ice at − 8°F to steam at 300°F, the specific heat of steam for the given conditions being 0.5.

559. How much steam at 100°C and 76 cm pressure will be required to melt 1,000 grams of ice at 0°C?

560. Assume the specific heat of ice to be 0.5 and that of steam at constant pressure to be 0.48. Find the result of putting in contact with each other 500 grams of ice at − 15°C, 400 grams of water at 50°C and 100 grams of steam at 120°C at atmospheric pressure.

561. Two hundred grams of water at 10°C are put into a copper cup weighing 100 grams. How much steam must be condensed in the water to heat it to 40°C when the pressure is 1 atmosphere?

562. How much heat is necessary to change 100 grams of ice at −20°C to steam at 110°C?

563. Find the result of mixing 40 grams of ice at 0°C with 40 grams of water at 35°C.

564. How much water at 75°C will have to be poured upon 1 kg of ice at −10°C in order that the resulting temperature be 20°C?

565. A mass of 1,000 grams of copper at 100°C is placed in a cavity in a block of ice. It remains in the cavity until it comes to the temperature of the ice. How many grams of ice are melted?

566. Find the temperature after mixing 10 lb. of water at 100°F, 10 lb. of alcohol (sp. h. 0.6) at 70°F, and 10 lb. of mercury (sp. h. 0.033) at 20°F, neglecting the thermal capacity of the containing vessel.

567. Assume that the mixture (problem 566) be made in a porcelain vessel (sp. h. = 0.15) having an initial temperature of 70°F. and a final temperature of 87°F. Find the mass of the containing vessel.

568. Find the specific heat of a given metal from the following data: 300 grams of the metal at 99°C are dropped into a hole in a block of ice, the temperature of which is 0°C. The hole is immediately covered with another block of dry ice. A total mass of 33.5 grams of ice is melted.

569. The specific heat of copper is 0.09. A copper calorimeter weighing 100 grams contains 200 grams of water at 10°C. 300 grams of copper at 100°C. are dropped into the water. Find the final temperature of the water.

570. A piece of copper, mass 120 grams, temperature 100°C, is dropped into 240 grams of water at 20°C contained in a metal cup of mass 300 grams. The resulting temperature is 23°C. Find (*a*) the specific heat of the cup; (*b*) its thermal capacity.

571. A piece of metal having a specific heat of 0.1, and a temperature of 90°C is placed in a cavity in a block of ice at 0°C. If 100 grams of ice are melted, what is the mass of the metal?

572. Ten pounds of steam at 212°F are mixed with 100 lb. of water at 70°F, and 2 lb. of ice at 22°F. Find the resulting temperature and condition of the mixture.

573. A piece of copper, mass 100 grams, temperature 96°C, is dropped into 240 grams of water at 20°C, contained in a zinc vessel of mass 300 grams. (*a*) Find the final temperature of the water; (*b*) the thermal capacity of the zinc calorimeter.

574. Three hundred grams of copper, temperature 100°C, are dropped into 400 grams of alcohol, temperature 20°C, contained in an aluminum vessel of mass 100 grams. Find the rise in temperature of the alcohol.

575. One hundred grams of ice at −10°C are dropped into a nickel calorimeter (specific heat 0.11) of mass 100 grams containing 500 grams of water at 30°C. Find the resulting temperature.

576. Find the specific heat of a crystal that is soluble in water but insoluble in turpentine, from the following data: The mass of the crystal is 45 grams. It is heated to 80°C and is then dropped into a copper calorimeter containing turpentine at a temperature of 22°C. Mass of calorimeter, 30 grams; mass of turpentine, 131 grams; sp. h. of turpentine, 0.43; final temperature of the turpentine, 30°C.

577. The burning of a pound of coal produces about 14,000 B.t.u. If this energy could be used in lifting coal out of a mine 100 ft. deep, how many tons of coal could be taken out of the mine for every pound burned?

578. The heat of combustion of a given grade of coal is 15,000 B.t.u. per lb. An engine and boiler furnishes 200 hp. while consuming 360 lb. of this coal per hr. Find the efficiency of the engine.

579. If we heat a quart of water (2 lb.) from 52°F to the boiling point how many foot-pounds of energy are put into it?

580. In burning 1 lb. of a certain coal 14,000 B.t.u. are developed. This is equivalent to how many calories per gram?

581. A given gas on combustion yields 560 B.t.u. per cu. ft. and costs $1 per 1,000 cu. ft. Find the cost of heating a gallon of water (8 lb.) from 72°F to the boiling point, assuming that the burner and kettle have a combined efficiency of 40 per cent.

582. How much will the air in a room 5 by 4 by 3 m be warmed due to the condensation of 1 kg of steam in the radiator, considering the density of air to be 0.00129 and its sp. h. 0.235?

583. The water of Niagara Falls descends about 50 m. Assuming that none of the heat is dissipated, find the increase of temperature per gram due to the fall.

584. Consider the average temperature of a lake to be 12°C. If the temperature fall to 10°C, how much heat per cubic centimeter would be given up in (*a*) calories? (*b*) B.t.u. per cubic foot?

585. If a tight vessel which neither absorbs nor transmits heat could be made, what would happen if 300 grams of ice at 0° and 50 grams of steam at 100° were put into it at the same time?

586. A lead bullet, mass 10 grams, strikes a target with a velocity of 500 m per sec. Assuming that 20 per cent. of the energy heats the bullet, what will be its rise in temperature?

587. The earth moves in its orbit nearly 19 miles per sec. or about 3,000,000 cm per sec. What is the heat equivalent of the energy of each gram of the earth's mass due to this motion?

588. (*a*) How many cubic feet of illuminating gas (Table XIX) will have to be burned in order to furnish the same amount of heat (B.t.u.) as that given by 1 ton of Scranton coal? (*b*) Compare the cost of the two, basing your estimate on the present rate in your town.

589. Analysis of a given grade of coal shows the following composition: C, 80 per cent.; H, 5 per cent.; O, 2 per cent. Find the value in B.t.u. (Table XX) of a ton of coal, assuming that all the carbon is burned to CO_2 and 95 per cent. of the hydrogen is burned to H_2O.

590. Assuming that dry wood (maple) weighs 1,200 lb. to the cord. (*a*) Find how many cords of wood will be required to furnish the same amount of heat as that given by 1 ton of Pocahontas coal (Table XX). (*b*) Compare the cost of the two at the prices which prevail in your community.

591. If the "conducting power" of brickwork be 4.8, how much heat will be conducted through a solid brick wall, 10 by 10 ft., 8 in. thick, in 8 hr., the difference in temperature on the two sides of the wall being 60°F.

592. Compare the quantity of heat conducted through the walls of a boiler on a winter's day when the temperature is 20°F and in summer when the temperature is 110°F, assuming that the temperature on the inside of the boiler is the same in both cases.

CHAPTER VII

ELECTRICITY

MAGNETISM

110. Magnetic Poles.—For convenience we shall speak of the north-seeking pole of a magnet as the N-pole, or N; the south-seeking pole, as the S-pole, or S.

Like poles repel; unlike poles attract.

A magnetic *pole* of *unit strength* is one which at a distance of 1 cm in a vacuum repels an equal and similar pole with a force of 1 dyne.

111. Coulomb's Law.—The magnitude and sense of the force of attraction or repulsion between two poles is given by Coulomb's law, which may be expressed in equational form as,

$$F = \pm\, mm'/\mu d^2$$

in which F = force in dynes; m and m' = the respective pole strengths; d = distance in centimeters between the poles m and m'; and μ = permeability of the medium. The sign + indicates repulsion between the poles m and m'; the sign −, attraction.

112. Permeability.—The *permeability* of a medium is a property which modifies the action of magnetic poles placed in the medium.

Permeability (μ) varies greatly with different media, and for a given medium, such as iron, it varies also with intensity of the magnetizing field. For a vacuum $\mu = 1$; for air at 20°C under a pressure of 1 atmosphere, $\mu = 1.000005$. In the case of iron the permeability may be as high as 2,000 and over. See Table XXVII.

113. Intensity of Magnetic Field.—The intensity of a magnetic field at a point may be measured by the force which it exerts on a unit pole at that point. In other words, magnetic field intensity H is force per unit pole; that is $H = F$ per unit pole $= \pm\,(m \times 1)/\mu d^2$, or the field intensity at a point p with reference to the pole m is

$$H = \pm\, m/\mu d^2.$$

The *unit of field intensity* is the gauss. A *gauss* is a field intensity of 1 dyne per unit pole.

Magnetic field intensity is a vector quantity, having the magnitude, direction, and sense of the force acting on unit positive pole.

Example.—Two magnetic poles, $m = +200$ units and $m' = -200$ units, are 16 cm apart. Find the magnitude, direction, and sense of the intensity of the field at a point p, 10 cm from each, Fig. 71.

Solution.—The magnetic intensity at p with respect to m is $H = +200/10^2 = +2$ *dynes per unit pole, the + sign indicating repulsion between m and p.*

The intensity at p with respect to m′ is $H = -200/10^2 = -2$ *dynes per unit pole, the* − *sign indicating attraction between m′ and p. The resultant of these two vector quantities, pa and pb is pc. The magnitude of pc is* 3.2; *its direction is parallel to the line joining m and m′; its sense is from p to c.*

114. Magnetic Moment.—A magnet makes an angle α with the direction of a magnetic field, Fig 72. The torque T in dyne centimeters acting upon the magnet is

$$T = Hml \sin \alpha = MH \sin \alpha.$$

The factor M is called the *magnetic moment;* it is the product of the strength of one of the poles into their distance apart; that is, $M = ml$.

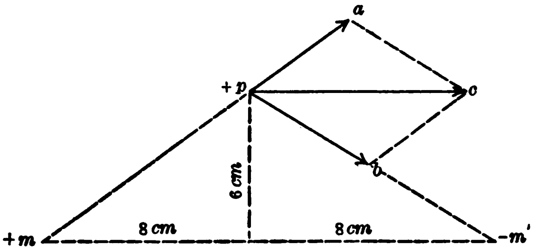


FIG. 71.—Magnitude, direction, and sense of magnetic field.

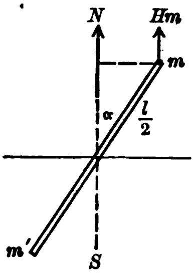


FIG. 72.—Magnetic moment.

115. Magnetic Induction and Magnetic Flux.—The magnetic field intensity H at a given point may be likened to a stress in an elastic body; the magnetic induction B may be likened to the corresponding strain. The relation of the magnetic induction B to the field intensity H, and to the permeability μ of the medium is expressed by the equation

$$\textit{Magnetic induction} = B = \mu H.$$

It is sometimes desirable to represent magnetic induction graphically by lines, called "lines of induction." In the case of a magnet, the lines of induction (sometimes called lines of force) are closed curves coming out of the N-pole and entering again at the S-pole. In this sense, induction B may be defined as the number of lines of magnetic induction per unit area.

Magnetic flux ϕ is the total number of lines passing through a given area A, that is,

$$\textit{Magnetic flux} = \phi = BA.$$

116. Terrestrial Magnetism.—The three important factors to be considered with reference to the earth's magnetism, for a given time and place, are (*a*) the magnetic declination, (*b*) the magnetic dip, (*c*) the intensity of the field. These three elements of terrestrial magnetism are of such great commercial and scientific importance that our Government, through the Coast and Geodetic Survey, and other authorized branches of service, maintains permanent stations for the collection and tabulation of data relating thereto.

Magnetic declination at a given place is the angle which the magnetic needle makes with the geographical meridian at that place. At Ann

Arbor, Mich., for example, the angle of declination is at present about 2° W. This means that the N-pole of the magnetic needle points to the west of true north by 2°. True north from a given point (in the northern hemisphere, for example) is the direction represented by a meridian connecting the point and the north geographic pole.

Magnetic dip is the angle which a dipping needle, when placed in the magnetic meridian, makes with the horizon. In Fig. 73 the angle of dip is designated by θ. At the magnetic equator $\theta = 0$; at the magnetic pole, $\theta = 90°$.

If for any given place we let the total intensity of the field, measured in the direction of the field, be H', then we may resolve this intensity into a horizontal component H, and a vertical component V, such that

$$H' = \sqrt{H^2 + V^2}$$

and the angle of dip θ, Fig. 73, may be determined by means of the equation

$$\tan\ \theta = V/H.$$

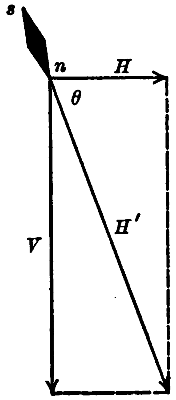


FIG. 73.—Magnetic dip.

The *horizontal component* of the earth's field H may be determined by means of a small bar magnet suspended at its midpoint so as to vibrate freely in the given magnetic field. The equation is

$$T = 2\pi\sqrt{I/MH}$$

where T = period of vibration of the magnet; I = moment of inertia of magnet; M = magnetic moment; H = horizontal component of the field.

Problems

NOTE.—In the solution of the following problems we shall assume, unless stated to the contrary, that the permeability of air is equal to unity, that pole strengths are expressed in c.g.s. units, and that Coulomb's law holds.

593. Explain the meaning of each term in the following equations. (*a*) $F = \pm mm'/\mu d^2$; (*b*) $H = \pm m/\mu d^2$; (*c*) $B = \mu H$; (*d*) $\tan\ \theta = V/H$.

594. How far from an N-pole of 10 units must an S-pole of 20 units be placed so that the attraction shall be 2 dynes?

595. Find the magnitude and sense of the intensity of the magnetic field (H) midway between the two poles of problem 594.

596. A magnetic needle of length 20 cm and magnetic moment (M) 200, lies in a magnetic field of intensity 2 gausses in such a position that the torque acting upon it is 100 dyne centimeters.

Find the angle that the magnet makes with the lines of the field.

597. The intensity of a magnetic field, the cross-sectional area of which is a rectangular figure 5 by 6 cm, is 2 gausses. The permeability of the medium is 5. Find (*a*) the induction B; (*b*) the flux ϕ.

598. At a certain place the angle of dip θ is 70°, and the horizontal component of the earth's field H is 0.19 gauss. Find the vertical component V.

599. A magnetic pole of 40 units acts with a force of 32 dynes upon another pole 5 cm away. Find strength of other pole.

600. Two magnetic poles of strengths 18 and 24 units respectively attract each other with a force of 3 dynes when placed in air. Find the distance between them.

601. Consider a right-angled triangle of base AB 8 cm, and altitude AC 6 cm. Assume that a magnetic pole of $+10$ units is placed at A; a pole of -40 units at B; and one of $+30$ units at C. Find the force (*a*) of attraction between A and B; (*b*) of repulsion between A and C; (*c*) the attraction between B and C.

602. Find the magnitude, direction, and sense of the resultant force at A, due to the action of the three poles of problem 601.

603. Find the magnitude, direction, and sense of the field H midway between A and B (problem 601).

604. Find the magnitude, direction, and sense of the field at A (problem 601), assuming that the $+10$ unit pole at this point has been eliminated.

605. Two equal bar magnets lie on opposite sides of a square, each side of which is 20 cm. The distance between the poles of each magnet is 20 cm. Make a drawing to illustrate the magnitude, direction, and sense of the resultant force when (*a*) the N-poles touch the same face; (*b*) when the N-poles touch opposite faces.

606. The poles of a given magnet are 16 cm apart, and lie at the extremities of the base of a right-angled triangle, the altitude of which is 12 cm. The pole strength (m) is 10 units. Find the magnitude of H at a point forming the third vertex of the triangle, and show by means of a sketch its direction and sense.

607. A given magnet of pole strength m lies on the diameter of a circle, the poles being on the circumference. Illustrate by diagram the magnitude, direction, and sense of H at a point on the circumference, 30° from the x-axis.

608. Consider two concentric circles having radii 10 and 20 cm respectively. A magnet, of pole strength 10, lies in the diameter of the smaller circle, its poles being on the circumference. Find the intensity of the field H at a point (*a*) where the x-axis cuts the larger circle; (*b*) where the y-axis cuts the larger circle.

609. What force must be applied to a magnet whose magnetic moment is 500 to hold it in an east and west position if the distance between the poles is 25 cm and H is 0.18 gauss?

610. A magnet is placed with its axis on the magnetic meridian and its south pole pointing north. It is found that there is a neutral point at a distance of 14 cm from the south pole of the magnet. The length of the magnet is 10 cm, and H is 0.18 c.g.s. units. Find the pole strength of the magnet.

611. Two equal bar magnets, each having a distance of 10 cm between the poles and a pole strength of 20 units, are placed with their axes in the same straight line and their poles pointing in the same sense, adjacent poles being 10 cm apart. Find the resultant force between the magnets.

612. Find the intensity of the field at a point halfway between the magnets of problem 611.

613. Two magnetic poles m and m' lie 20 cm apart. The pole strength of m is $+12$ units. Find a point on a line joining the poles where the field intensity H is zero, (*a*) when m' is $+3$; (*b*) when m' is -3.

614. At Ann Arbor the vertical component of the earth's field (V) is 0.54 gauss, and the angle of dip 71° 30′. Find the magnitude of the earth's magnetic force H' at this place, and (*b*) its horizontal component.

615. A magnet of length 12 cm, pole strength 20, is suspended by a fine wire so as to rest in a horizontal plane and in the magnetic meridian. The horizontal component of the magnetic field is 0.18 gauss. When the upper end of the wire is twisted through 90°, the magnet is deflected 30° from the meridian. Find the torque tending to restore the magnet to the meridian.

616. Two places A and B, are 10 miles apart, as measured on a magnetic meridian, which, between these places, is a straight line. The angle of declination at A is 2°. C is on the same parallel of latitude as *B* and is directly north of *A*, as measured on a geographical meridian. Find the distance from *B* to *C*, measured in feet.

617. According to the 1910 report of the U. S. Great Lakes' Survey the horizontal component of the earth's field at the lower end of Lake Michigan was 0.1871 gauss, and the angle of dip 72° 28′. Find the vertical component of the earth's field from these data?

618. In Ann Arbor the horizontal component of the earth's field (H) is 0.18 gauss; the vertical component (V) 0.54 gauss. Find (*a*) the angle of dip θ; (*b*) the magnitude of the earth's magnetic field (H') at this place.

619. It was found that a magnet suspended horizontally at Bristol, England, made 110 complete vibrations in 5 min., and that the same magnet at St. Helena made 112 vibrations in 4 min. Find the ratio of the values of H at the two places.

620. The magnetic dip at Bristol is 70°; that at St. Helena, 80°. Find the total force of the earth's magnetism at St. Helena, that at Bristol being 0.48.

621. A long bar magnet *AB* is suspended in a horizontal position about its middle point. The pole strength of this magnet is 100 units. A second long magnet *ab* is placed vertically beneath the end *A* of the first magnet, so that the distance *Aa* is 2 cm. The poles *A* and *a* attract. A 2.5-gram weight attached to *B* is required to keep the bar *AB* in equilibrium. Find the pole strength of *a*.

622. A magnet suspended to vibrate in a horizontal plane is caused to oscillate in two places. At the first it makes 100 vibrations in 5 min.; at the second place, 110 vibrations in 5 min. Compare the values of the horizontal components of the earth's magnetic fields at the two places.

623. A magnet makes 15 oscillations per min. in a certain magnetic field. How many will it make per minute if re-magnetized so that its magnetic moment is increased 50 per cent.?

CURRENT RESISTANCE. FALL OF POTENTIAL

117. Magnetic Field Due to a Circular Current.—If a conductor carrying a current be bent into a loop or coil, of radius r, the loop will have the property of a magnet; the face of the loop into which the magnetic lines enter being the S-pole, and the side or face out of which the lines come being the N-pole. In the case of a circular current, where we consider the proportionality factor between H and I to be unity, the intensity of the magnetic field H at the center of the coil is

$$H = 2\pi NI/r = 2\pi NI'/10r$$

in which H = intensity of field in gausses; N = number of turns of wire in the coil; I = current in c.g.s. units; I' = current in practical units (amperes); r = radius of coil in centimeters.

118. Units of Current Strength.—The electrical units of current strength, quantity, resistance, and fall of potential, are, in general, defined in three ways; that is, as (*a*) c.g.s. units, (*b*) practical units, and (*c*) international units.

C.G.S. Unit of Current Strength.—The equation $H = 2\pi NI/r$ gives us a means of defining I in terms of H and r as follows: The electromagnetic c.g.s. unit of current is that current which, flowing through an arc of unit length, in a circle of unit radius, will produce a unit magnetic field at the center of the circle.

Practical Unit.—The practical unit of current strength is the ampere. An ampere is 10^{-1} c.g.s. units.

International Unit.—A very close approximation to the ampere, as defined above, may be made experimentally by means of the silver coulometer. This unit is known as the international ampere, which is defined as that unvarying current which will deposit silver from a standard solution of silver nitrate at the rate of 0.00111800 gram per sec.

The ampere (10^{-1} c.g.s. units) and the international ampere are so nearly identical that no discrimination is ordinarily made between these units.

119. Units of Quantity.—The *electromagnetic c.g.s. unit of quantity* is the quantity transferred by 1 c.g.s. unit in 1 sec.; that is,

$$Q = It$$

where I and t represent c.g.s. unit values.

The *practical unit of quantity* is the *Coulomb*, which is the quantity of electricity transferred by 1 amp. in 1 sec. One coulomb = 10^{-1} c.g.s. unit of quantity.

Problems

624. Given a circular coil of five turns, of mean radius 2 cm, carrying a current of 10 c.g.s. units. Find (*a*) the intensity of the field at the center of the coil due to the current; (*b*) the force in dynes with which this field will repel a 5-unit positive magnetic pole.

625. Assume that a current of 10 amp. flows through the coil (problem 624). Find (*a*) the current strength in c.g.s. units; and (*b*) the intensity of the field at the middle of the coil due to the current; (*c*) the force in dynes with which a 10-unit magnetic pole will be urged along the axis.

626. What must be the radius of a circular loop of wire such that the field due to a current of 20 amp. will repel a +5-unit pole at the center of the coil with a force of 20 dynes?

627. A circular coil of wire CC', Fig. 74, lies with its vertical face in the plane of the earth's field. Assume that the horizontal component of the earth's field is 0.18 gauss; that the needle is short as compared with diameter of coil; number of turns in the coil around the needle, 10; radius of the coil, 10 cm; the angle α, 30°. Find the current in amperes flowing through the coil.

628. A uniform current deposits from a standard silver nitrate solution 5.031 grams of silver in half an hour. Find the current in (*a*) amperes; (*b*) c.g.s. units.

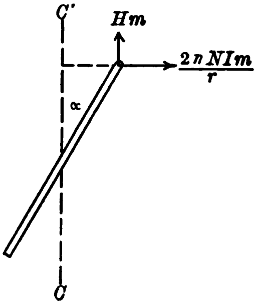


FIG. 74.—Magnetic forces at center of a coil.

629. What quantity of electricity is required to deposit 0.6708 gram of silver from a standard $AgNO_3$ solution in 20 min.? Give your results in (*a*) practical units; (*b*) c.g.s. units.

120. Resistance.—Electrical resistance is that property of a conductor by virtue of which the energy of a current is transformed into heat. Resistance R is defined in terms of the equation $W = RI^2t$, in which W is the heat energy in ergs generated by I c.g.s. unit of current in t seconds; and R is the resistance in c.g.s. units.

121. Units of Resistance.—*C.G.S. Unit of Resistance.* The c.g.s. unit of resistance is that resistance by which heat equal to one erg is produced per second by unit current.

Practical Unit.—The *ohm* is 10^9 c.g.s. units of resistance.

International Ohm.—The international ohm is a near approximation to the ohm (10^9 c.g.s. units) as determined by the following specifications: "The international ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of constant cross-sectional area, and of length of 106.300 cm."

In the practical application of these units no distinction is ordinarily made between the ohm (10^9 c.g.s. units) and the international ohm.

122. Laws of Resistance.—The resistance of a conductor is a function of its length, cross-sectional area, kind of material, and temperature.

Metric Units.—For a given temperature, the resistance R in ohms is

$$R = kl/a$$

where l = length in centimeters; a = cross-sectional area in square centimeters; and k is the resistivity (specific resistance) in ohm-centimeters.

Resistivity may be defined as the resistance in ohms of a conductor having a cross-sectional area of 1 cm², and a length of 1 cm.

Resistivity Tables, see page 200.

English Units.—For a given temperature, the resistance R in ohms is

$$R = kl/d^2$$

where l = length in feet; d = diameter in mils; and k = resistance in ohms per mile-foot. A "mil" is a thousandth of an inch; a "circular mil" (d^2) is a mil squared; a "mil-foot" represents a conductor 1 ft. in length and 1/1000 in. in diameter.

For values of k in ohms per mil-foot, see Tables, page 200.

Example.—Find the resistance of 50 m of aluminum wire, the radius of which is 0.5 mm.

Solution.—Referring to the Resistivity Tables, page 200, *we find that* k *for aluminum* $= 2.6 \times 10^{-6} = 0.0000026$ *ohm-cm. The area* $a = \pi r^2 = \pi(5/100)^2 = 25\pi/10000$. *Then* $R = 0.0000026 \times 50 \times 100 \times 10000/25\pi = 5.2/\pi$ *ohms.*

Example.—Find the resistance of 1 mile of copper wire, the diameter of which is 0.03 in.

Solution.—From Table XXIV we find that k (*ohms per mil-foot*) *for copper is* 9.5. *The length of* $l = 5{,}280$ *ft. Diameter* $d = 0.03 = 30/1000 = 30$ *mils; hence* $d^2 = 900$. *Then* $R = kl/d^2 = 9.5 \times 5280/900 = 55.7$ *ohms.*

123. Change of Resistance with Temperature.—In general the resistance of a metallic conductor *increases* with increase of temperature, in accordance with the equation $R_t = R_o(1 + \alpha t)$, where α is the temperature coefficient of resistance. For pure metals α is nearly 0.004 per degree C. The temperature coefficient (α) for alloys is very much lower than that of pure metals.

The resistance of carbon and electrolytes *decreases* with increase of temperature.

124. Resistances in Series and in Parallel.—Resistance of conductors in series and in parallel may be expressed in equational form as,

$$\textit{Series,} \quad R = R' + R'' + \ldots\ldots$$
$$\textit{Parallel,} \quad 1/R = 1/R' + 1/R'' + \ldots.$$

125. Conductance and Conductivity.—*Conductance* is the reciprocal of resistance; *conductivity* (specific conductance) is the reciprocal of resistivity. The terms conductance and conductivity are, in general, applied to electrolytes.

Example.—The resistance of a cylindrical column of $CuSO_4$ solution, of cross-section 2 cm², and length 6 cm, is 60 ohms. (*a*) What is the conductance? (*b*) the conductivity?

Solution.—(*a*) *Conductance* $= 1/R = 1/60$ *reciprocal ohms.* (*b*) *The resistivity* (*specific resistance*) $= 60/(2 \times 6) = 5$ *ohms-cm. The conductivity, then,* $= \frac{1}{5}$ *reciprocal ohm.*

Problems

630. Find the resistance of 20 m of iron wire, diameter 0.4 mm, at 18°C.

631. Compare the resistance of 4 m of German silver wire, radius 0.2 mm, with that of 2,700 cm of nickel wire, radius 0.3 mm.

632. What length of copper wire, of radius 0.5 mm, will be required to furnish a resistance equal to that of 15 m of manganin wire, of radius 0.2 mm?

633. Find the resistance of 10 miles of iron wire having a diameter of 150 mils.

634. A copper wire is 10 ft. in length; its diameter is 0.025 in. What is (*a*) its diameter in mils? (*b*) its resistance in ohms?

635. Find the resistance of 6 m of pure platinum wire, B. & S. gage No. 40, at (*a*) 32°F; (*b*) 32°C. (See Wire Gage Tables, page 200.)

636. Find the resistance of 5 m of No. 30 copper wire at 18°C.

637. What will be the resistance of the wire of problem 636 if both its length and diameter be doubled?

638. Find the resistance of the wire (problem 636) at (*a*) 0°C; (*b*) 30°C.

639. Find the resistance of a mile of a given wire having a diameter of 0.02 in., the resistance (k) of a mil-foot being 60 ohms.

640. The resistance of No. 12 copper wire having a diameter of 0.08 in. is 1.6 ohms per 1,000 ft. Find the resistance of No. 00 trolley wire, having a diameter of 0.365 in.

641. The resistance of a cube of a given sample of copper 1 cm on each edge, density 8.9 grams per cm^3, is 1.6 microhms. Find the resistances of 1.78 kg of this sample of copper when drawn into wire, B. & S. gage, No. 20.

642. A wire is stretched uniformly until its length is doubled. Compare its resistance before and after stretching.

643. Three conductors have resistances of 2, 4, and 6 ohms respectively. Find the resistance of the system when connected in (*a*) in series; (*b*) in parallel.

644. Enumerate all the resistances that can be obtained from three coils of resistances 2, 4, and 6 ohms respectively, by the various ways in which they may be connected, all three coils being always in use.

645. The specific resistance of copper is 0.00000156 ohm at 0°C. Find the resistance of a copper wire 1 mm in diameter and 100 m in length, if its temperature is 30°C.

646. The two circular, parallel platinum electrodes of a conductivity cell are 0.5 cm apart. The cross sectional area of each electrode is 1 cm^2. With this cell it was found that at 20°C the resistance of a tenth normal (n/10) solution of KCl between the electrodes was 42.92 ohms. Find (*a*) the conductance of the n/10 KCl solution; (*b*) the conductivity.

647. Find the area of the electrodes of a conductivity cell in order that the resistance of an n/10 KCl solution be 64.38 ohms, when the electrodes are set 3 cm apart, temperature and other conditions to be the same as in problem 646.

126. Difference of Potential and Electromotive Force.—The electric energy of a current may be expressed as $W = I^2Rt = IR \times It = IR \times Q$, from which we may write $IR = W/Q$. That is, the expression IR may be defined as the work done in carrying unit quantity of electricity through a resistance R. The product IR measures the IR drop, or *difference of potential,* due to a current I flowing through a resistance R. We may therefore write

$$\textit{Potential difference} = V - V' = IR.$$

The *e.m.f.* of a system is the *IR drop around the entire circuit.*

Example.—A battery having an internal resistance of 4 ohms, delivers a current of 0.5 ampere through an external resistance R of 16 ohms. Find (*a*) the fall of potential over the external circuit; (*b*) fall of potential over the internal circuit; (*c*) the e.m.f. of the battery.

Solution.—(*a*) $IR = 0.5 \times 16 = 8$ *volts;* (*b*) $Ir = 0.5 \times 4 = 2$ *volts;* (*c*) e.m.f. $= I(R + r) = 0.5 \times 20 = 10$ *volts.*

127. Ohm's Law.—One of the most important generalizations in electricity is that known as Ohm's law, which holds for constant currents, and which may be stated in equational form as

$$I = E/R, \text{ or } E = IR.$$

This law implies that when E and R are once fixed, the current strength is the same at every point throughout the circuit, the conductors of which are in series. Thus for a given current containing a fixed E and R, Fig. 75, the current strength, as registered by ammeters A and A', will be the same at both A and A'. If the circuit contains resistances in parallel, Fig. 76, the current $I = I' + I''$.

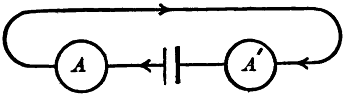


FIG. 75.—Current of equal strength throughout the circuit.

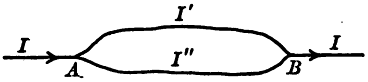


FIG. 76.—Current in a divided circuit.

128. Fall of Potential over Conductors in Parallel.—A problem of special importance to the student is that of the fall of potential over conductors in parallel, especially as it furnishes a method of determining the current strength in the various branches of the conductors in the parallel system.

The important point to note in this connection is this: ***The fall of potential over each branch of a parallel system from A to B***, Fig. 76, ***is the same as the fall of potential over the entire system from A to B.*** If R be the resistance of the parallel circuit of which R' is the resistance of one branch, and R'' the resistance of another branch, then $E = IR = I'R' = I''R''$.

Example.—Two conductors of 4 and 12 ohms respectively are connected in parallel to the points A and B. A current of 4 amp. flows from A to B, a part passing through each conductor. Find (a) the resistance from A to B; (b) the fall of potential from A to B; (c) the current through each conductor.

Solution.—(a) $1/R = 1/4 + 1/12 = 1/3$. *Hence* $R = 3$ *ohms.* (b) *Fall of potential from* A *to* $B = E = IR = 4 \times 3 = 12$ *volts.* (c) $I' = 12/4 = 3$ *amp.*; $I'' = 12/12 = 1$ *amp.* *Verification:* $I = I' + I'' = 3 + 1 = 4$ *amp.*

129. Units of Potential Difference. *C.G.S. Unit.*—The c.g.s. unit of potential difference is the difference of potential produced at the terminals of unit resistance when traversed by unit current.

Practical Unit.—The practical unit of potential difference is the volt, which is equivalent to 10^8 c.g.s. units.

International Volt.—The international volt is the electric pressure which, when steadily applied to a conductor whose resistance is an international ohm, will produce a current of an international ampere.

The cadmium or *Weston standard cell,* when set up according to standard specifications has an e.m.f. of

$$E_t = 1.0183 - 0.00004(t - 20) \text{ volts.}$$

130. The E.M.F. of Some Commonly Used Cells.—It is frequently convenient to know the e.m.f. of some of the cell types that are in common use to-day. It must be noted that the following e.m.fs. are only approximate working values.

Name of cell	E.m.f.
Daniell cell	1.1 volts
Leclanché cell	1.5 volts
Dry cell	1.4 volts
Lead storage cell	2.1 volts
Edison storage cell	1.2 volts

131. Cells in Series and in Parallel.—In connection with this subject, it is important to bear in mind that the e.m.f. of a cell depends only upon (a) the nature of the electrolyte, (b) the kind of electrodes used, and (c) the temperature, and that it is entirely independent of the size of the plates. It is also important to note, on the other hand, that the internal resistance of a battery is *inversely* proportional to the size of the plates.

Cells in Series.—The current I for n cells in series, where R is the external resistance and r is the internal resistance of one cell, is

$$I = nE/(R + nr).$$

Cells in Parallel.—The current I for n cells in parallel, where E is the e.m.f. of a single cell, is

$$I = E/(R + r/n).$$

Multiple Series.—Cells may be connected in multiple series, as shown in Fig. 77, in which n is the number of cells in each series and m is the number of series. For multiple series the current is

$$I = nE/(R + nr/m)$$

in which R = external resistance; and nr/m = internal resistance of the system. It is evident that I will be a maximum when the denominator of the fraction is a minimum. It may be shown that $R + nr/m$ is a minimum when $R = nr/m$. It should be noted, however, that the conditions for maximum current in multiple series, as given above, do not hold if there is a counter e.m.f. in the circuit; nor does it hold for a grouping of cells for quickest action when there is an e.m.f. of self-induction in the circuit.

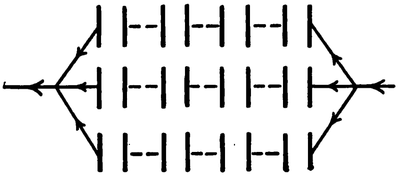


FIG. 77.—Cells in multiple series; $n = 4$, $m = 3$.

132. Thermoelectromotive Force.—*Thermoelectromotive force* (t.e.m.f.) is the e.m.f. due to a difference of temperature at the juncture of two substances. For example, if a copper iron juncture be heated so that the difference in temperature between A and B, Fig. 78, is 200°C there will be developed in the system an e.m.f. of about 0.0027 volt, directed at the hot juncture from the copper to the iron.

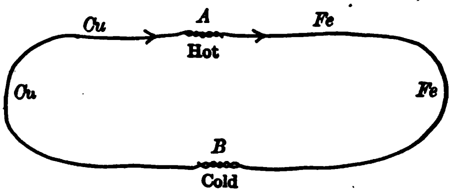


FIG. 78.—Thermoelectromotive force.

Thermoelectric power is the variation of thermoelectromotive force per degree change in temperature. For Thermoelectric Power Tables, see page 200.

Example.—Reference to Table XXVI, reveals the fact that the thermoelectric power of iron is +17.5 microvolts per degree C, and that of German silver, −12 microvolts. Find the thermoelectromotive force of 20 German silver-iron couples in series, the difference of temperature between junctures being 80°C.

Solution.—Since the value for iron (+17.5) *is on the* positive side of the zero point in the series, and the value for German silver (−12) is on the negative side, the total difference between the two values is 29.5 microvolts = 0.0000295 volt per degree. The total thermoelectromotive force then is $E = 0.0000295 \times 80 \times 20 = 0.0472$ volt.

Problems

648. Consider Fig. 79, in which R is 4 ohms, R' is 6 ohms, and R'' is 12 ohms. The current flowing through the ammeter A is 2 amp. (*a*) What is the total resistance between B and C? (*b*) What is the fall of potential (IR drop) from B to C? (*c*) What is the IR drop over each conductor?

649. Consider Fig. 80, when R is 4 ohms, R' is 6 ohms, and R'' is 12 ohms, as in problem 648. The current through the ammeter A is 4 amp. (*a*) What is the total resistance from B to C? (*b*) The IR drop from B to C? (*c*) The IR drop over each conductor? (*d*) The current through each conductor?

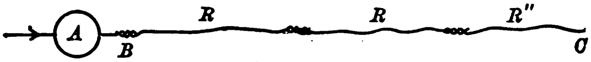


FIG. 79.

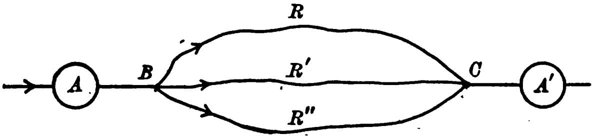


FIG. 80.

650. Given three conductors A, B, and C, having resistances of 2, 3, and 6 ohms respectively, and a battery, having an e.m.f. of 7 volts and an internal resistance of 3 ohms. Find (*a*) the current flowing through the battery when A, B, and C are connected in series across the terminals; (*b*) the IR drop over each conductor.

651. The conductors A, B, and C (problem 650) are in parallel. Find (*a*) the current through the battery; (*b*) the IR drop over each conductor.

652. The current flowing through the ammeter, A, Fig. 80, is 5 amp. The resistance of R is 2 ohms; R' is 5 ohms; R'' is 8 ohms. Find (*a*) the resistance from B to C; (*b*) the IR drop from B to C; (*c*) the fall of potential over each one of the three lines; (*d*) the current through each line (R, R', R''); (*e*) the current through A'.

653. Three resistances, 50, 100, 150 ohms, are connected in series to a generator, the terminal potential of which is 150 volts. Find (*a*) the current through each resistance; (*b*) the drop of potential over each.

654. Assume that the three resistances (problem 653) are connected in parallel. Find the current through each conductor.

655. Given five conductors, A, B, C, D, E, having resistances of 1, 2, 3, 4, 5 ohms respectively. The five conductors are connected in series to a battery, the internal resistance of which is negligible. The current through the system is 2 amp. Find (*a*) the IR drop over each conductor; (*b*) e.m.f. of the battery.

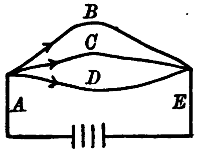


FIG. 81.

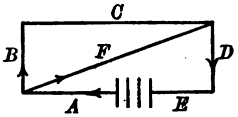


FIG. 82.

656. Consider the five conductors of problem 655 to be connected in parallel from terminal to terminal. Find (*a*) the resistance of the circuit; (*b*) the IR drop in each wire.

657. Consider Fig. 81. The resistance of A is 2 ohms; B, 3 ohms; C, 4 ohms; D, 5 ohms; E, 2 ohms. The e.m.f. of the battery is 10 volts. Find (*a*) the fall of potential over A; (*b*) the fall of potential over B, C, D; (*c*) the IR drop over E.

658. The resistances of the various conductors of Fig. 82 are as follows: A and E are 2 ohms apiece; B and D are 4 ohms apiece; C is 5 ohms; and F is 8 ohms. The e.m.f. of the battery is 12 volts. Find (*a*) the fall of potential over C; (*b*) over F.

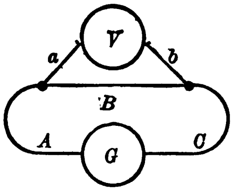


FIG. 83.

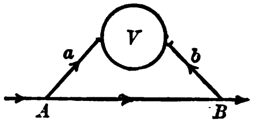


FIG. 84.

659. Three wires, A, B, C, each having a resistance of 100 ohms respectively, are connected in series across the terminals of a generator G, Fig. 83, of constant e.m.f. of 580 volts, and negligible internal resistance. A voltmeter of resistance 900 ohms is connected across the terminals of B. Find the reading of the $V.M.$, the resistance of a and b being negligible.

660. Consider Fig. 84. The resistance from A to B is 10 ohms. The resistance of the voltmeter is 100 ohms. The resistance of the connecting wires is negligible. A current of 5 amp. flows through *AB*. Find (*a*) the fall of potential from *A* to *B*; (*b*) the voltage from *A* to *B*; (*c*) the fall of potential over the voltmeter.

661. (*a*) What current flows through the voltmeter of problem 660? (*b*) What current flows through the circuit beyond *B*? (*c*) If an ammeter of a negligible resistance were put in the circuit to the left of *A*, what would its reading be?

662. A storage battery of constant e.m.f., consisting of five cells, has a total internal resistance of 0.5 ohms. Connected to its terminals is a wire of 10 ohms of resistance. The current flowing through the wire is 1 amp. Find (*a*) the terminal potential (fall of potential over external circuit); (*b*) internal drop of potential; (*c*) e.m.f. (fall of potential around entire circuit).

663. Consider conditions of problem 662. A second wire of 10 ohms resistance is put in from terminal to terminal of the battery parallel with the first wire. (*a*) What is the resistance of the two wires in parallel? (*b*) What current flows through the battery? (*c*) What is the drop of potential over the external circuit?

664. A battery, e.m.f. 20 volts, internal resistance 0.2 ohms, has connected across its terminals a wire of 100 ohms resistance. A voltmeter of resistance 490 ohms is connected to this wire at two points *A* and *B* the resistance between *A* and *B* being 10 ohms. Find the reading of the *V.M.*

665. If the *V.M.* (problem 664) be removed, (*a*) what will be the value of the current in the wire? (*b*) the fall of potential over *AB*?

666. Two wires each of 10 ohms resistance are connected in parallel to the terminals of the battery of problem 664. Find (*a*) the current in each wire; (*b*) the current through the battery; (*c*) the *IR* drop around the entire circuit.

667. The two wires of problem 664, are connected in series to the terminals of the battery. Find (*a*) the *IR* drop over each wire; (*b*) the internal drop of potential.

668. A battery having an internal resistance of 5 ohms has a wire *ABC* connected at its terminals. The resistance of *AB* is 4 ohms and that of *BC* 6 ohms. A current of 2 amp. flows through the circuit. Find (*a*) the fall of potential over the external circuit; (*b*) the e.m.f. of the battery.

669. A voltmeter having a resistance of 100 ohms is connected to the points AB, problem 668. Find (*a*) the resistance of the entire circuit; (*b*) the current through BC; (*c*) reading of the voltmeter.

670. Given three conductors, A, B, C, having resistances of 1, 3, and 5 ohms respectively. These three conductors are connected in series to a battery, the internal resistance of which is 1 ohm. The current through the system is 2 amp. Find (*a*) the IR drop over each conductor; (*b*) the internal drop of potential; (*c*) the e.m.f. of the battery.

671. Consider the three conductors of problem 670 to be connected in parallel from terminal to terminal of the battery. Find (*a*) the resistance of the circuit; (*b*) the IR drop over each conductor.

672. Five dry cells, each having an e.m.f. of 1.5 volts, and an internal resistance of 0.1, 0.2, 0.3, 0.4, 24 ohms respectively, are connected in series. (*a*) What current will the five cells furnish through an external resistance of 5 ohms? (*b*) What current will be furnished through the same resistance if the last cell be cut out?

673. A storage cell, e.m.f. = 2.1, $r = 0.1$ and a Daniell cell, e.m.f. = 1.1, $r = 2.9$, are set in opposition. Find the current furnished through a resistance of 7 ohms.

674. Suppose that the voltmeter, Fig. 84, has a resistance of 94 ohms and the connecting wires, a and b, have a resistance of 2 ohms each. The resistance of AB is 2 ohms. The voltmeter registers 18.8 volts. Find (*a*) the current through the voltmeter; (*b*) the current in a and b; (*c*) the fall of potential from A to B; (*d*) the current over AB.

675. A conductor $ABCD$ is connected in series to the poles of a generator of constant e.m.f., the terminal potential of which is 360 volts. Resistance AB is 200 ohms; BC, 50 ohms; CD, 200 ohms. Find (*a*) the current through the system; (*b*) the fall of potential over BC.

676. A voltmeter having a resistance of 200 ohms is connected to the points BC (problem 675). Find (*a*) the current in AB; (*b*) the fall of potential from B to C; (*c*) the reading of the voltmeter; (*d*), the current through the voltmeter.

677. Suppose that the points BC (problem 675) be connected directly to the poles (360 volts). The resistance of BC is, as before, 50 ohms. Find the fall of potential over BC (*a*) with the voltmeter out; (*b*) voltmeter in.

678. An arc lamp, requiring 5 amp. to operate it, resistance 15 ohms, is connected in series with a resistance of 5 ohms, across the terminals of a generator, internal resistance 2 ohms. Find (*a*) the voltage across the lamp; (*b*) the brush potential of the generator; (*c*) the e.m.f. developed by the generator.

679. A current of 2 amp. flows through a series circuit containing a generator of resistance 1 ohm, line of 2 ohms, lamp 97 ohms. Find (*a*) the e.m.f. developed by the generator; (*b*) its brush voltage.

680. A battery, e.m.f. 20 volts on open circuit, internal resistance 5 ohms, is delivering a current of 2 amp. Find (*a*) the external resistance; (*b*) the terminal potential.

681. Calculate the resistance of a galvanometer shunt in terms of the resistance of the galvanometer if it is desired to have one-fiftieth of the current go through the galvanometer.

682. A galvanometer having a resistance of 5,000 ohms is shunted with 100 ohms. A certain deflection of the galvanometer is obtained with a battery of constant e.m.f. when the resistance of the rest of the circuit is 2,000 ohms. What additional resistance must be inserted to produce the same deflection when the shunt is removed?

683. Consider Fig. 85. Assume that the bridge is in balance; that is, the fall of potential over CD is zero. (*a*) How does the fall of potential over AC compare with that over AD? (*b*) How does the current over AC compare with that over AD? (*c*) How does the IR drop over CB compare with that over AC? (*d*) How does the current over CB compare with that over AC?

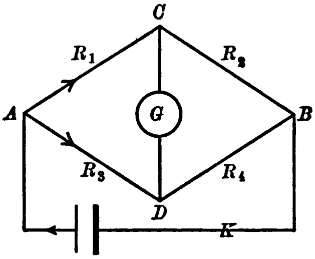


FIG. 85.

684. Consider five conductors ($A = 2$ ohms, $B = 4$ ohms, $C = 6$ ohms, $D = 8$ ohms, $E = 10$ ohms), Fig. 86, to be connected to a battery of e.m.f. 50 volts, internal resistance and the resistance of a and b negligible. Find (*a*) the resistance of the circuit; (*b*) the IR drop over each wire.

685. Consider the five conductors (problem 684) to be connected to a battery of 30 volts e.m.f., as shown in Fig. 87. (*a*) Find the IR drop in B, C, and D respectively. (*b*) How does the fall of potential over $B + C$ compare with that over E?

686. A battery having an e.m.f. of 88 volts and internal resistance of 2 ohms, furnishes current through a circuit consisting of a 4-ohm coil and a 6-ohm coil in parallel. Find (*a*) the current in the battery; (*b*) the current in each coil; and (*c*) the potential difference between the extremities of the coils.

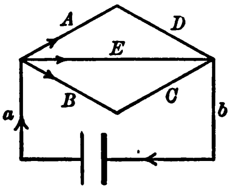


FIG. 86.

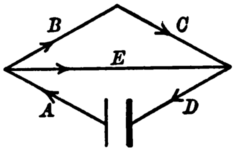


FIG. 87.

687. With an external resistance of 8 ohms a given cell furnishes a current of 0.14 amp. With an external resistance of 5 ohms it gives a current of 0.2 amp. Find the internal resistance of the cell.

688. The potential difference between the terminals of a cell is 1.5 volts when the cell is furnishing no current. When its terminals are connected by a wire of 2 ohms resistance, the terminal potential falls to 1.2 volts. Find its internal resistance.

689. Fifty incandescent lamps in parallel constitute a group at a distance of 200 ft., from a center of distribution at which the voltage is 110 volts. Each lamp takes 0.5 amp. Find the maximum resistance of conductors allowable if a 2 per cent. drop in line is specified as the maximum drop.

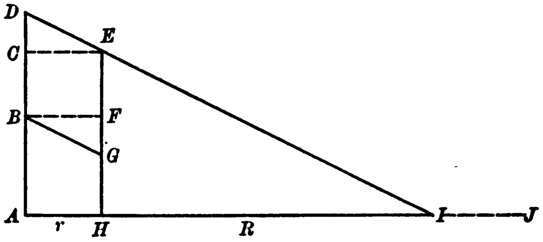


FIG. 88.

690. In problem 689, what would be the voltage at the lamps if only 10 lamps were lighted?

691. In Fig. 88 we have a diagrammatic representation of the e.m.f., terminal potential, internal resistance, and external resistance of a cell, in which AD is the e.m.f.; AC, the terminal potential; r and R the internal and external resistances respectively. Suppose that the external resistance R be increased from I to J. Make drawing to show how this will affect (*a*) the e.m.f. of the cell (AD); (*b*) the internal IR drop (FG). (*c*) Will the internal resistance of the battery r be affected? (*d*) What external resistance R will be required in order to make the external IR drop (AC) equal to the e.m.f. of the cell?

692. Given four cells, the e.m.f. of which is 1.5 each, and the internal resistance 2 ohms each. Find the current furnished through a resistance of 100 ohms when the cells are connected in (*a*) series; (*b*) parallel. Solve for conditions (*a*) and (*b*) when the external resistance is 1 ohm.

693. How should four cells be connected to produce maximum current through a 2-ohm coil, if each cell has an e.m.f. of 1.6 volts, and an internal resistance of 2 ohms? What is the maximum current?

694. Given five cells, each having an internal resistance of 0.5 ohm and an e.m.f. of 1.2. Find the current through an external resistance of 75 ohms when the cells are connected in (*a*) series; (*b*) parallel.

695. Find the current through a 0.5 ohm coil (cells as in problem 694) when connected in (*a*) series; (*b*) parallel.

696. Given 12 cells each having an internal resistance of 0.5 ohm and an e.m.f. of 2 volts. Make diagrams to illustrate six arrangements of these cells (series, parallel, multiple series).

697. Find the current furnished by each arrangement (problem 696) through an external resistance of 10 ohms.

698. How must a battery of 10 cells, each having a resistance of 2 ohms and an e.m.f. of 1.1 volts, be arranged to give the largest current through an external resistance of 5 ohms? Find the value of the current.

699. Three cells each having a resistance of 2 ohms and e.m.f. of 1.1 volts are to be used to produce current in a wire of 2 ohms resistance. Diagram three ways of connecting them, and compute the current in each case.

700. Given a battery of 12 cells, the e.m.f. of each cell being 1 volt, and its internal resistance 4 ohms. Make sketch to illustrate set-up for maximum current for an external resistance of 3 ohms. Calculate the current strength.

701. Given 10 cells, 1 volt each, internal resistance 3 ohms each. Find the current through a resistance of 10 ohms when the cells are connected in (*a*) series; (*b*) parallel.

702. Find the current through an external resistance of 1 ohm (cells as in problem 701) in (*a*) series; (*b*) parallel.

703. Find the thermoelectromotive force of an iron-German silver couple, the difference in temperature between the two junctures being 130°C. See Tables, page 200.

704. A difference of 50°C between the faces of a antimony-bismuth thermopile gives a thermoelectromotive force of 0.2034 volt. Of how many elements (antimony-bismuth) is the thermopile composed?

705. One juncture of an iron-constantan couple consisting of a single element is placed in ice; the other juncture in an oil bath, the temperature of which is 55°C. Find the thermoelectromotive force generated.

706. An iron-constantan thermopile consists of 100 couples, the resistance of each couple being 0.1 ohm. The two faces of the instrument are maintained at a difference of temperature of 200°. What current will the thermopile furnish through an external resistance of 2 ohms?

707. It is desired to install a copper-constantan thermopile in a chimney in such a way that one set of junctures is within the chimney, and the other set outside. The average difference of temperature maintained between the two faces of the system is 95°C. How many elements will be required to furnish an e.m.f. as great as that of a dry cell (e.m.f. = 1.34)?

708. If we consider that each of the two wires (copper-constantan) of each element is 1 ft. in length, and that No. 10 wire was used, what current will the thermopile of problem 707 furnish through an external resistance of 5 ohms?

709. The maximum effective area of the working face of a given thermopile is A; its length from face to face is L. It is desired to draw a maximum current from this thermopile through a given external resistance R, the temperature difference being T. By using wire of diameter d it is possible to have 16 junctures in each face of the pile; by using wire of diameter $d/2$ it is possible to have 32 junctures per face. Which number of junctures will furnish the greater current for a given R and T?

ELECTROLYTIC AND HEATING EFFECTS OF A CURRENT. ELECTRIC POWER

133. Dissociation and Electrolysis.—Acids, bases and salts when placed in water undergo, to a certain degree, dissociation into positive and negative ions. The following reactions illustrate a few typical cases of dissociation in water.

HCl in water $= \overset{+}{H} + \overset{-}{Cl}$

KCl in water $= \overset{+}{K} + \overset{-}{Cl}$

HNO_3 in water $= \overset{+}{H} + \overset{-}{NO_3}$

$AgNO_3$ in water $= \overset{+}{Ag} + \overset{-}{NO_3}$

H_2SO_4 in water $= \overset{+}{H} + \overset{+}{H} + \overset{--}{SO_4}$

Na_2SO_4 in water $= \overset{+}{Na} + \overset{+}{Na} + \overset{--}{SO_4}$

$CuSO_4$ in water $= \overset{++}{Cu} + \overset{--}{SO_4}$

$ZnCl_2$ in water $= \overset{++}{Zn} + \overset{-}{Cl} + \overset{-}{Cl}$

$NaOH$ in water $= \overset{+}{Na} + \overset{-}{OH}$

KOH in water $= \overset{+}{K} + \overset{-}{OH}$

134. Chemical Equivalents.—The *chemical equivalent* of a substance is its atomic weight divided by its valence. The valence of the radicals given in the preceding article is in each case indicated by the number of charges (positive or negative) which the radical carries. For example, the valence of H and of Ag is 1; the valence of Cu and of Zn is 2, and so on.

The following table gives the symbol, atomic weight, valence, and chemical equivalent of a number of elements. For the sake of convenience in computation, values are given to the first decimal place only.

Symbol	At. W.	Valence	Chemical Eqv.
H	1.0	1	1.0
O	16.0	2	8.0
Cl	35.5	1	35.5
K	39.1	1	39.1
Na	23.0	1	23.0
Ag	107.9	1	107.9
Cu	63.6	2	31.8
Zn	65.4	2	32.7

A *faraday* is the quantity of electricity ($Q = It$) required to liberate a chemical equivalent (gram ion) of a given substance. A faraday = 96,500 coulombs. This means that the passage of 96,500 coulombs through an electrolytic cell will liberate 1 gram of H, 8 grams of O, 35.5 grams of Cl, 107.9 grams of Ag, and so on.

135. Electrochemical Equivalent.—The *electrochemical equivalent* of a substance is the mass in grams deposited by a current of 1 amp. flowing for 1 sec. By international agreement the electrochemical equivalent of silver is 0.001118 gram per coulomb. With the electrochemical equivalent of silver as a starting point, we may compute from the chemical equivalent of a given substance its electrochemical equivalent.

Example.—The electrochemical equivalent of Ag is 0.001118 and the chemical equivalent is 107.88. Find the electrochemical equivalent of H, its chemical equivalent being 1.

Solution.—Electrochemical equivalent of Ag: electrochemical equivalent of H = chemical equivalent of Ag: chemical equivalent of H. Then $0.001118 : x = 107.88 : 1$. *Hence electrochemical equivalent of H* $= x = 0.00001036$.

136. Faraday's Laws of Electrolysis.—In terms of the data given in the preceding articles, Faraday's laws may be stated thus:

I. The mass of a substance deposited electrolytically is equal to the product of its electrochemical equivalent times the current in amperes, times the time in seconds. The law may be written $M = ZIt$, where M is the mass in grams deposited, Z is the electrochemical equivalent of the substance in question, I is the current in amperes; t is the time in seconds.

II. If the same quantity of electricity pass through different electrolytes, the masses liberated at the electrodes are proportional to the chemical equivalents.

Example.—A given quantity of electricity passes through two electrolytic cells, one containing $AgNO_3$ and the other containing $CuSO_4$. Two grams of silver are deposited in the one cell. How many grams of copper (Cu) will be deposited in the other cell?

Solution.—$M : M' = C.E. : C'.E'$. *Then* $5 : M' = 107.9 : 31.8$. *Hence* $M' = 1.47$ grams.

Problems

710. Define: Chemical equivalent, electrochemical equivalent, faraday. What is the chemical equivalent of hydrogen? oxygen? How many grams of H will be liberated by 2 faradays? How many grams of O?

711. (*a*) What is the atomic weight of silver? its valence? chemical equivalent? electrochemical equivalent? (*b*) How much Ag will be deposited by 1 faraday?

712. How many grams of zinc will be consumed in 10 hr. by a battery delivering 2 amp.?

713. What time in seconds will be required to refine by electrolysis 5 kg of copper by means of a current of 10 amp.?

714. (*a*) What time will be required to deposit 1 gram of silver electrolytically with a current of 2 amp.? (*b*) How much copper will be deposited by the same current in the same time?

715. What time will be required to decompose electrolytically 90 grams of water by a current of 2 amp.? How many grams of oxygen will be liberated?

716. In a certain test it was found that a current of 4.5 amp. would decompose 18 grams of water in 12 hr. From these data compute the electrochemical equivalent of oxygen?

717. A current passes through three electrolytic cells in series; one contains a solution of silver nitrate, the second a solution of copper sulphate, and the third acidulated water. It is found that 2.7 grams of silver are deposited. Calculate the mass of copper, hydrogen, and oxygen liberated.

718. Two electrolytic cells each containing copper sulphate and having a resistance that is very high as compared with all other resistances in the circuit are placed first in series then for the same length of time in parallel. Compare the total quantities of salt decomposed in the two cases.

719. (*a*) How long will a current of 0.5 amp. have to flow to deposit 0.1 gram of silver on a metal spoon? (*b*) How much hydrogen will the current liberate in the same time?

720. How much will a metal plate be increased in weight if it be nickel-plated by a current of 0.5 amp. running 5 hr.?

721. A Daniell cell furnishes 0.1 amp. for 1 hr. (*a*) How much zinc goes into solution? (*b*) How much copper is deposited on the positive electrode?

722. A deposit of 6.445 grams of Cu is made by a current flowing for 1 hr. through a copper coulometer. What quantity (coulombs) of electricity passed through the cell?

723. A copper coulometer and a silver coulometer are placed in series. What is the ratio of the deposits in the two coulometers?

724. An ammeter is calibrated by means of a silver coulometer, the ammeter and coulometer being in series. A constant current is passed through both instruments for 1 hr. The reading of the ammeter during this time is 0.76 amp.; the amount of silver deposited in the platinum bowl of the coulometer is 3.0186 grams. Find (*a*) the error in the ammeter reading; (*b*) the percentage of error.

137. Electric Energy Expended as Heat.—The energy expended by a current in the form of heat is proportional to the square of the current and the time which it flows; that is,

$$Energy = I^2Rt = EIt.$$

When I, R, and t are given in c.g.s. units, the energy is expressed in *ergs*. When, however, I is the current in amperes, R is ohms and t is seconds, then the energy is expressed in *joules*.

Since 1 calorie = 4.186 × 10^7 ergs = 4.186 joules, and consequently 1 joule $= \frac{1}{4.186} = 0.24$ cal., then

$$Amperes^2 \times ohms \times seconds \times 0.24 = calories.$$

138. Electric Power.—As formulated in the preceding article, *electrical energy* $= I^2Rt = EIt$. Power is the expenditure of energy per unit of time; that is

$$Power = I^2R = EI.$$

When I, R, and E are given in c.g.s. units, power is expressed in *ergs per second.* When I is amperes, R is ohms, and E is volts, then power is expressed in *joules per second,* that is, in *watts.*

$$\text{Amperes}^2 \times \text{ohms} = \text{volts} \times \text{amperes} = \text{watts}.$$

139. Watt-hours. Kilowatt-hours.—The total amount of energy expended by a current may be measured (*a*) in terms of the amount of metal deposited in an electrolytic cell; (*b*) in terms of the quantity of heat generated; and (*c*) in terms of power multiplied by the time. The last-named method is the one most usually employed in measuring the total energy expended by a current. The units are the *watt-hour,* or more commonly the *kilowatt-hour.* A kilowatt-hour is 1,000 watt-hours.

$$\text{Watt-hours} = \text{watts} \times \text{time in hours}.$$

Example.—A current of 0.5 amp. flows through an incandescent lamp for 5 hr. Find (*a*) the power expended in watts; (*b*) the energy expended in watt-hours; (*c*) in kilowatt-hours; (*d*) in ergs; (*e*) joules.

Solution.—(*a*) $EI = 110 \times 0.5 = 55$ *watts.* (*b*) $55 \times 5 = 275$ *watt-hr.* (*c*) $275/1000 = 0.275$ kw-hr. (*d*) $55 \times 10^7 \times 5 \times 60 \times 60 = 99 \times 10^{11}$ *ergs.* (*e*) $99 \times 10^{11}/10^7 = 99 \times 10^4$ *joules.*

Problems

725. A 110-volt 16-cp. lamp takes 0.45 amp. of current. Find the heat in calories developed per minute.

726. A current of 5 c.g.s. units flows through a conductor for 2 min. The energy expended in heat is 24 joules. Find (*a*) the energy expended in ergs; (*b*) the resistance of the conductor in c.g.s. units; (*c*) in ohms.

727. What power, in watts, is used by a conductor, (*a*) which takes 2 amp. at 110 volts? (*b*) which takes 2 amp. through a resistance of 55 ohms?

728. A car heater, supplied with a pressure of 550 volts, uses a current of 5 amp. Find (*a*) the resistance of the heater; (*b*) calories developed per hour.

729. How much energy, in joules, is used by a 55-watt lamp burning 10 min.?

730. A bank of incandescent lamps takes 10 amp. at 110 volts, 80 per cent. of the energy received being given off as heat. Find the heat given off in 1 hr. in (*a*) joules; (*b*) calories.

731. Two wires, A and B, of the same material and length are connected in series. The diameter of A is to that of B as 2:1. The resistance of A is 2 ohms. The applied e.m.f. is 10 volts. Find the heat in calories generated in B during 10 sec.

732. A 16-cp. carbon filament incandescent lamp on a 110-volt circuit takes 0.5 amp. Find its efficiency in watts per candlepower. At 10 cts. per kw-hr. for electric energy, what is the cost per hour of operating this lamp?

733. An ordinary gas jet consumes about 6 cu. ft. of gas per hr. for 18 cp.; a gas mantle burner, 4 cu. ft. per hr. for 60 cp. At $1 per 1,000 cu. ft. of gas, compare the cost per candlepower-hour of these lamps with the incandescent lamp of problem 732, electric energy being furnished at 10 cts. per kw-hr.

734. An arc lamp takes a current of 10 amp. at 110 volts. Its candlepower is about 2,200. Find its efficiency in watts per candlepower.

735. An electrolytic cell for the purification of copper has a resistance of 0.01 ohm. How much heat will be developed in this cell per minute at a time when 10 grams of copper are being deposited per minute?

736. The arc lamp for street lighting takes 480 watts. If the current used is 6.4 amp., what voltage must be applied to a circuit operating 60 lamps in series allowing 2 per cent. voltage drop in the lines?

737. Suppose that we have a battery of five storage cells. A wire of 10 ohms resistance is connected to the terminal of the battery. The internal resistance of the battery is 2 ohms. A current of 0.8 amp. flows through the circuit. Find (*a*) the drop of potential (the *IR* drop) over the external circuit; (*b*) the drop of potential over the internal circuit; (*c*) the e.m.f. of the battery. (*d*) Find the heat developed in the wire in 10 min. in joules; ergs; calories.

738. A 40-watt tungsten incandescent lamp is capable of giving about 32 cp. Compare the efficiency of this lamp in watts per candlepower with that of a 16-cp. carbon filament lamp which takes 0.5 at 110 volts.

739. Consider Fig. 84, p. 119. The resistance of *AB* is 100 ohms, that of the voltmeter circuit 1,000 ohms. The current flowing across *AB* is 2 amp. Find the power expended on *AB* in (*a*) watts; (*b*) kilowatts. (*c*) Find the heat in calories developed in *AB* in 10 min.

740. Find the total energy expended on the voltmeter (problem 739) in 10 min. in (*a*) ergs; (*b*) joules.

741. Three wires *AB*, *AC*, *CB*, have resistances of 5, 3, and 2 ohms respectively. These conductors are connected in the form of a triangle *ABC*, of which *AB* is the base. An e.m.f. of 10 volts is applied to the points *A* and *B*. Find the total heat expended on the system in calories in half an hour.

742. Consider the three wires of problem 741 to be connected in series, and an e.m.f. of 100 volts to be applied to the ends of the wire. Find (*a*) the power expended in each wire in watts; (*b*) the total energy expended on the system in 10 sec. (*c*) Find the heat developed in each wire in calories during a period of half an hour.

743. A current of 1.2 amp. under a pressure of 110 volts flows through an incandescent lamp for 5 hr. (*a*) Find the power expended in watts. (*b*) Find the energy expended in watt-hours. (*c*) Find the total energy expended in ergs.

744. A 220-volt motor is using 10 amp. of current. It is geared to an apparatus for lifting coal from a mine. The efficiency of the system is 60 per cent. About how long a time will be required for this apparatus to lift a ton of coal from a mine 200 ft. deep.

745. If it requires 100 hp. to drive a dynamo producing 137 amp. at 500 volts terminal potential difference, find the efficiency of the machine.

746. Find the horsepower required to operate 150 incandescent lamps, each taking 0.45 amp. at 110 volts.

747. Find the current used by an 8-hp. 500-volt motor if its efficiency is 90 per cent.

748. Find the current used if 100 kw. are to be transmitted (*a*) at 110 volts; (*b*) at 2,200 volts.

749. For the same weight of copper conductors and the same percentage drop in the lines, compare the distances to which the same power could be transmitted, using the two voltages 110 and 2,200, problem 748.

750. For the conditions given in problem 749, compare the heat losses in the lines.

751. A storage battery has an e.m.f. of 60 volts and an internal resistance of 0.8 ohm. When it is being charged a current of 20 amp. is used. (*a*) What is the applied e.m.f., and (*b*) what is the rate at which energy is stored in the battery?

752. Assuming an efficiency of 50 per cent., what current would be used by a 220-volt electric hoist when raising 2,200 lb., at the rate of 30 ft. per min.?

CHAPTER VIII

ELECTRICITY (Continued)

MAGNETIC EFFECTS OF A CURRENT

140. Magnetic Field.—When an electric current flows through a conductor there is set up in the medium around the conductor lines of magnetic induction. The direction and sense of these lines may be determined by the right-hand rule, which is stated as follows: *Grasp the conductor with the right hand, the thumb pointing in the direction of the current, and the fingers will indicate the sense of the lines of magnetic induction,* Fig. 89. If, on the other hand, we consider that we are looking at the end of the conductor, the right-hand rule again gives us the sense of the magnetic field. In Fig. 90 there is represented a conductor, viewed "end-on," in which the current is represented as flowing into the paper; the lines of magnetic induction are clockwise in sense. In Fig. 91 the current is represented as coming out; the field is counter-clockwise in sense. It should be noted that a cross, Fig. 90, representing the feathered end of an arrow, indicates an "*in*" current; a dot, Fig. 91, representing the point of the arrow, indicates an "*out*" current.

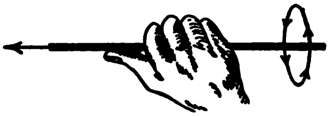


Fig. 89.—Thumb indicates sense of current; figures, sense of magnetic field.

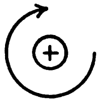


Fig. 90. "In" current.

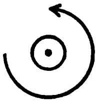


Fig. 91. "Out" current.

141. Deflection of Magnetic Needle.—A magnetic needle in a magnetic field always tends to set itself in such a position that the lines of induction of the field enter the S-pole and come out of the N-pole. Suppose that a wire carrying a current is placed in a north-south position directly above a magnetic needle. The needle is now acted upon by two fields, the earth's field in a north-south direction, and the field due to the conductor, at right angles to the earth's field. The position of the needle is such that the respective torques due to the two fields are equal to each other.

Problems

753. Draw a line to represent a magnetic needle, the N-pole being to the right. Draw a second line to represent a conductor carrying a current flowing from A to B. Make drawings now to indicate the direction of deflection of the needle when (*a*) the conductor *AB* is above the needle; (*b*) below the needle; (*c*) beside the needle, and between it and the observer.

754. A wire carrying a current lies in a north-south direction. Find the sense of the current for the following deflections of the N-pole of the needle; (*a*) needle above the wire, N-pole to the east; (*b*) needle beside the wire, N-pole down.

755. A circular coil of wire carrying a current lies in a north-south vertical plane. On which side of the coil (east or west) must an N-pole be placed in order that repulsion occur?

756. A circular conductor lies in the plane of the paper. Determine by the right-hand rule the polarity of the face of the coil toward the observer when the current is (*a*) in a counter-clockwise sense; (*b*) clockwise.

142. Magnetic Effect of a Solenoid.—The magnetic intensity in the interior of a solenoid whose length is great in comparison with its cross-section, or of a solenoid bent into the form of a closed ring, may be shown to be

$$H = 4\pi nI = 4\pi nI'/10$$

in which I = current strength in c.g.s. units; I' = current strength in amperes; n = number of turns per centimeter length of the solenoid; and H = intensity of the field in gausses.

143. Magnetic Induction.—Magnetic induction B is equal to the permeability μ multiplied by the intensity of the magnetizing field H; that is, $B = \mu H$, from which we may write $\mu = B/H$.

A study of the magnetization curve of a given sample of iron, Fig. 92, reveals the fact the μ is not a constant, but varies from point to point, being greatest at the point of tangency of the line *OC* with the curve.

144. Magnetic Flux.—Magnetic induction B may be considered as the number of lines of magnetic induction passing through a substance per unit area. Magnetic flux ϕ through a given area A is the total number of lines passing through the area; that is, $\phi = BA$.

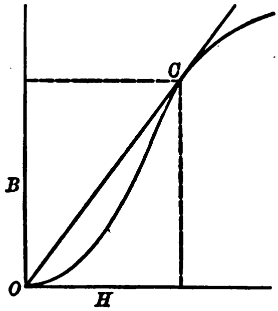


FIG. 92.—Magnetization curve, showing point of maximum permeability.

Example.—A helix 1 m. long containing 2,000 turns, and having a cross-sectional area of 10 cm², carries a current of 10 amp. The length of the helix is sufficiently great so that the end effects may be ignored with reference to the field near its middle point. The value of μ is 5. Find (*a*) the intensity of the field H near the middle of the helix; (*b*) the induction B; (*c*) the flux ϕ.

Solution.—(a) *The number of turns per centimeter length.* $n = 2{,}000/100 = 20$. $H = 2\pi \times 20 \times 10/10 = 40\pi$ *gausses.* (b) $B = 5 \times 40\pi = 200\pi$. (c) $\phi = 200\pi \times 10 = 2000\pi$ *lines of induction.*

145. Magnetization Curves.—In Fig. 93 we have plotted a number of magnetization curves for different samples of iron.

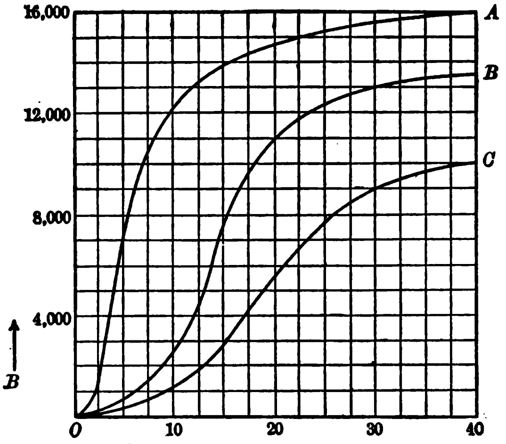


Fig. 93.—Magnetization curves.

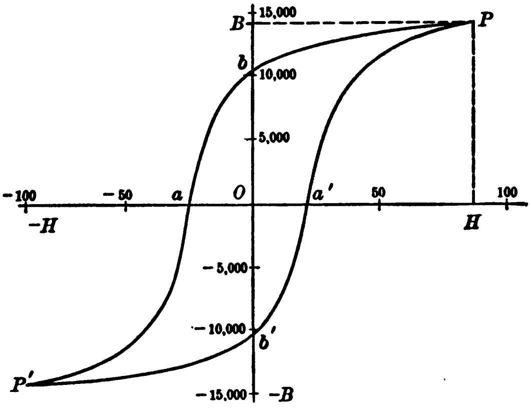


Fig. 94.—Hysteresis curve.

146. Magnetic Hysteresis.—*Hysteresis* is a lagging of the induction behind the magnetizing field. A hysteresis curve, Fig. 94, is a loop mapped out during a complete cycle of the field intensity, increasing from O to H, decreasing from H to O; then reversing the sense and increasing from O to $-H$, decreasing from $-H$ to O; reversing and increasing from O to the point when the loop is completed.

The *remanence* is the induction B which remains in the iron when the field H is reduced to zero at O. The *coercive force* is the negative field intensity required to reduce the induction to zero, as at a.

Problems

757. From Fig. 93, find the permeability μ for (*a*) curve A, for $H = 40$; (*b*) curve B, $H = 20$; (*c*) curve C, $H = 30$.

758. Since B/H is the tangent of an angle, show that μ will be a maximum for that point on the magnetization curve when a straight line drawn from O touches the curve tangentially on its upper side.

759. Consider the hysteresis curve, Fig. 94, (*a*) What is the remanence at the point b? (*b*) What is the coercive force at a?

147. Magnetomotive Force.—If a magnetic pole m be carried once around the magnetic circuit within a closed solenoid, against the magnetic field, W units of work will be done. *Magnetomotive force* (m.m.f. = Ω) is numerically equal to the work done in carrying a unit pole once around the magnetic circuit; that is, m.m.f. = W/m. If we let l be the length of the magnetic circuit, n the number of turns of wire per unit length, and N the total number of turns ($n = N/l$), then since $W = Fl$, and $F = Hm$, we may write

$$M.M.F. = W/m = 4\pi NI = 4\pi NI'/10,$$

where I = current in c.g.s. units, and I' = current in amperes.

The c.g.s. unit of m.m.f. is the *gilbert*, which is equal to 1 erg per unit pole. The practical unit of magnetomotive force is the *ampere-turn*. The product NI = *ampere-turns*, where N is the number of turns in the coil, and I is in c.g.s. units.

148. Magnetic Reluctance.—In the case of a magnetic circuit we have an equation which is very similar to Ohm's law ($I = E/R$) for electrical circuits,

$$\phi = M.M.F./\mathcal{R}$$

in which ϕ = magnetic flux ($\phi = BA$); m.m.f. = magnetomotive force; and $\mathcal{R}$ = *magnetic reluctance.*

The c.g.s. unit of flux is the *maxwell;* the c.g.s. unit of reluctance is the *oersted.*

Let μ be the permeability of the medium, l the length of the magnetic circuit, and A its cross-sectional area. Then the magnetic reluctance $\mathcal{R}$ is

$$\mathcal{R} = l/\mu A$$

Example.—An insulated wire is wrapped in the form of a spiral around a circular iron ring, the mean length of which is 60 cm and the cross-sectional area is 4 cm². The number of turns of wire per centimeter is 3. A current of 5 amp. flows through the wire of the solenoid. The permeability μ of the iron for the given conditions is 400. Find (*a*) the intensity of the magnetic field H; (*b*) the magnetic induction B; (*c*) the flux ϕ; (*d*) the magnetic reluctance $\mathcal{R}$; (*e*) the magnetomotive force m. m. f., by two methods.

Solution.—(*a*) $H = 4\pi \times 3 \times 5/10 = 6\pi$ *gausses.* (*b*) $400 \times 6\pi = 2{,}400\pi$, *lines of induction / cm²*. (*c*) $\phi = 2{,}400\pi \times 4 = 9{,}600\pi$ *lines.* (*d*) $\mathcal{R} = 60/(400 \times 4) = 0.0375$. (*e*) *M.m.f.* $= 4\pi NI = \phi\mathcal{R} = 360\pi$ *ergs/unit pole.*

ELECTROMAGNETIC INDUCTION

149. Induced E.M.F.—If a magnet be thrust into a coil of wire, an induced e.m.f. is set up in a given sense; if the magnet be withdrawn an e.m.f. is set up in an opposite sense. In general, whenever a conductor cuts across the lines of induction of a magnetic field, an induced e.m.f. is set up in the conductor. It is important to know the relation which exists between the motion of the conductor or field and the direction and sense of the induced e.m.f. There are a number of rules which may be employed to advantage to determine the direction and sense of the induced e.m.f. two of which are:

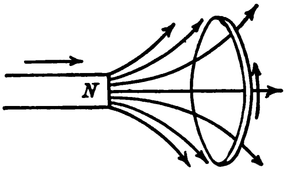


FIG. 95.—Induced e.m.f. in coil.

Rule I.—For the case of a circular conductor, the following rule is serviceable: Consider that the observer is looking in the direction and sense of the field. An *increase* of the number of lines threading through the coil gives an *indirect* (counterclockwise) e.m.f., Fig. 95; a *decrease* in the number of lines gives a *direct* (clockwise) e.m.f.

Rule II.—The right-hand rule, which was employed to determine the direction and sense of the magnetic field about a conductor, may be used also to determine the sense of the induced e.m.f.

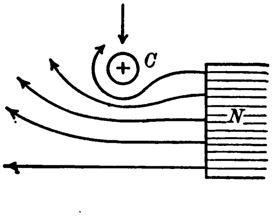


FIG. 96.—"In" e.m.f.

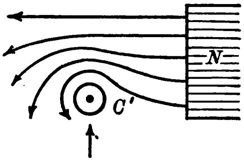


FIG. 97.—"Out" e.m.f.

In Fig. 96 we have an end view of a metal rod falling vertically through a magnetic field. Each line on being cut by the conductor C may be conceived of as tending to wrap itself around the rod. *Grasp the conductor with the right hand, the fingers being in the direction and sense of the lines of induction, and the thumb will extend in the direction and sense of the induced e.m.f.* According to this rule the e.m.f. in the conductor C, Fig. 96, is directed *into* the paper; the e.m.f. in C', Fig. 97, is directed *out.*

Problems

760. By means of the right-hand rule determine the direction and sense of the induced e.m.f. when the conducting rod R, Fig. 98, moves in the direction (a) from R to A; (b) from R to B; (c) R to C; (d) R to D; (e) R to E; (f) R to F.

761. Find the direction and sense of the induced e.m.f. when the conductor moves in the circular path (a) from A to B; (b) C to D; (c) at what two points does the sense of the induced e.m.f. change?

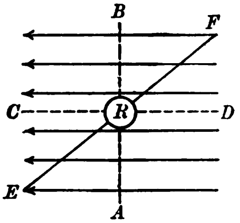


FIG. 98.

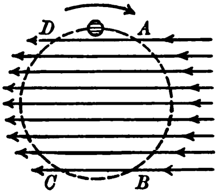


FIG. 99.

150. Magnitude of Induced Electrical Quantities.—Quantative experiments have shown that the relation of the induced electromotive force E to the number of turns N of the conductors and the time rate of change of the flux is

$$E = -N(\phi - \phi')/t \text{ c.g.s. units} = -N(\phi - \phi')/(t \times 10^8) \text{ volts},$$

where N = the total number of turns in the coil, the minus sign indicating that the induced e.m.f. is opposed to the action producing it; $\phi - \phi'$ = the change of flux in the time t seconds. Special note should be made of the fact (a) that E represents *average* values of the e.m.f. during the time t, and (b) that $(\phi - \phi')/t$ may represent either a time rate of change of flux through a given coil, or a time rate of cutting of lines of induction in the magnetic field.

Example.—A given coil of wire consisting of five turns, encloses 1,000 lines of induction. The flux changes from 1,000 to 200 in 0.1 sec. Find (a) the time rate of change of flux; (b) the average E in c.g.s. units; (c) in volts.

Solution.—(a) $(\phi - \phi')/t = 800/0.1 = 8{,}000$ *per sec.* (b) $E = 5 \times 8{,}000 = 40{,}000$ *c.g.s. units*; (c) $E = 40{,}000/10^8 = 0.0004$ *volt.*

Example.—A single straight conductor 1 m in length is moved across a magnetic field at right angles to the direction of the lines of induction with a uniform speed of 1 m in 2 sec. The induction B of the field is 800. Find the average E induced in the conductor in (a) c.g.s. units; (b) in volts.

Solution.—$\phi = BA = 800 \times 100 \times 100 = 8{,}000{,}000$ *lines of induction and,* $t = 2$. *Then* (a) $E = 8{,}000{,}000/2 = 4{,}000{,}000$ *c.g.s. units;* (b) $E = 0.04$ *volt.*

151. Counter E.M.F. of Self-induction (Lenz's Law).—When a current in changing (increasing or decreasing) there is set up in the circuit at every instant a counter e.m.f. of self-induction, which tends to oppose the change.

This opposition to change in an electromagnetic system is formulated by Lenz's law, which may be stated as follows: *When a change occurs in an electromagnetic system, that thing happens which tends to oppose the change.* Thus, if an N pole is thrust into a closed circuit, Fig. 100, the induced current will be of such a nature as to produce in the coil an N-pole opposing the motion of the magnet; and in a like manner, when the magnetic pole is withdrawn the inductive system opposes the change.

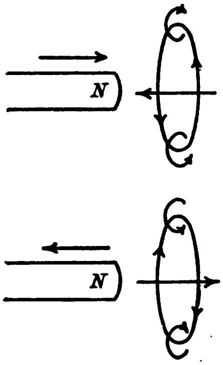


Fig. 100.—Lenz's law.

152. E.M.F. of Self-induction.—The e.m.f. of self-induction is expressed by the equations

$$E = -N(\phi - \phi')/t \text{ c.g.s. units} = -N(\phi - \phi')/(t \times 10^8) \text{ volts}$$

where E = e.m.f. of self-induction; N = total number of turns in the coil; $\phi - \phi'$ = change of flux in t sec.

153. Self-inductance.—The flux ϕ in a given coil multiplied by the number of turns N of the coil is called the *coil flux*, $N\phi$. Now the coil flux is proportional to the current I flowing in the coil, and by introducing the proportionality factor L we may write $N\phi = LI$, where L is the *coefficient of self-induction*, or the *inductance.*

Starting with the equation $N\phi = LI$, we may define L in terms of two equations as follows: First, by differentiating with respect to t, we have $-Nd\phi/dt = Ldi/dt$, from which

$$E = -N(\phi - \phi')/t = -L(I - I')/t$$

We may define L, then, as the ratio of the induced e.m.f. to the time rate of change of current.

Second, by letting $N = nl$, where n is the number of turns per unit length of the coil and l is its length, and by substituting $\phi = BA = \mu HA = 4\pi\mu nAI$, a second expression for inductance L may be obtained, as follows,

$$L = 4\pi\mu n^2lA \text{ c.g.s. units} = 4\pi\mu n^2lA/10^9 \text{ henrys.}$$

154. Mutual Inductance.—The mutual inductance of two coils is the ratio of the e.m.f. induced in one of the coils to the time rate of change of current in the other. The equation is

$$E = -Mdi/dt$$

in which E = induced e.m.f. in one coil; di/dt = time rate of change of the current in the other coil; and M = coefficient of mutual inductance.

155. Units of Inductance.—Self-inductance L and mutual inductance M are physical quantities of the same nature, and hence the same unit may be used for both.

The *c.g.s. unit of inductance* is an inductance such that a change of one c.g.s. unit of current per second will give rise to one c.g.s. unit of e.m.f.

The *practical unit* of inductance is the *henry*, which is equal to 10^9 c.g.s.

units of inductance. The henry is an inductance such that a current changing at the rate of 1 amp. per sec. will give rise to an induced e.m.f. of 1 volt.

Example.—In a coil the inductance of which is 5×10^6 c.g.s. units, a current of 12 amp. drops to 2 amp. in $\frac{1}{10}$ sec. Find the induced e.m.f. developed in volts.

Solution.—$L = 5 \times 10^6/10^9 = 1/200$ *henry.* $E = L(I - I')/t = (1/200)(12 - 2)/(\frac{1}{10}) = 0.5$ *volt.*

Example.—A helical coil of wire of cross-sectional area 4 cm^2 bent into the form of a ring of mean radius $60/\pi$ cm, consists of 3 turns per centimeter. Find the inductance of this coil (*a*) in c.g.s. units and (*b*) in henrys, when it is filled with a substance having permeability of 200.

Solution.—$L = 4\pi\mu n^2 lA = 4\pi \times 200 \times 9 \times 120 \times 4 = 3{,}456{,}000\pi$ *c.g.s. units* $= 0.003456\pi$ *henry.*

156. Energy Stored in the Field.—If we assume that in a circuit of self-inductance L the current rises from zero to I in t sec., then the average current during the time is $I/2$, and the quantity of electricity flowing through the circuit is $Q = It/2$. Also, since the time rate of change of the current (di/dt) is uniform, the induced e.m.f. is constant during the time t, and $E = LI/t$. Now the energy stored in the system during the time t (that is, while the current is rising from zero to I) is represented by $EQ = (LI/t)(It/2) = LI^2/2$, and hence we may write

$$EQ = W = LI^2/2$$

where W = energy stored in the circuit of inductance, L due to the rise of the current from zero to I. W is expressed in *ergs* when L and I are in c.g.s. units, and in *joules* when L is in henrys and I is in amperes.

Problems

762. Write the following equations, and explain the meaning of each term contained therein; $H = 4\pi nI$; $B = \mu H$; $\phi = BA$; $M.M.F. = 4\pi NI$; $\mathcal{R} = l/\mu A$; $\phi = M.M.F./\mathcal{R}$; $E = -N d\phi/dt = -L di/dt = -M di/dt$; $L = 4\pi\mu n^2 lA$.

763. State Lenz's law, and make drawings to illustrate application of the principle.

764. How would you determine from the curve B, Fig. 93, the maximum value for μ? Illustrate by a sketch.

765. A solenoid of cross-sectional area 5 cm^2, having 1,200 turns of wire, and bent in the form of a ring of mean radius 10 cm, carries a current of 5 amp. Consider the permeability of the air within the coil to be unity. Find (*a*) the intensity of the field H within the coil; (*b*) the induction B; (*c*) the flux θ.

766. Suppose that the solenoid of problem 765 is filled with an iron core of permeability 500, for the given conditions. Find (*a*) H; (*b*) B; (*c*) θ; (*d*) $\mathcal{R}'$; (*e*) m.m.f.

767. A current flows around a coil of wire in a counter-clockwise sense. Make a sketch of the coil illustrative of the direction and sense of the induced magnetic field within the coil (*a*) while the current rises to its maximum; (*b*) while the current remains steady; (*c*) while it falls to zero. Make sketch to illustrate direction and sense of the induced e.m.f. in the coil for the conditions (*a*), (*b*) and (*c*).

768. A current of 5 amp. flows through a solenoid consisting of 500 turns of wire, and bent in the form of a ring having a mean radius of r cm. Find the radius r such that H shall equal 50 gausses.

769. Consider Fig. 93. Compute from the curve marked C values of μ for field strengths (H) as follows: (*a*) 10; (*b*) 20; (*c*) 30; (*d*) find the maximum value of μ.

770. Five hundred turns of wire are wrapped around a metal ring, having a mean radius of 8 cm; cross-sectional area, 10 cm^2; permeability, 1,000. A current I flows in the solenoid, such that the field intensity (H) is 4π gausses. The maximum magnetic flux (θ) through this coil is reduced to zero in 0.1 sec. Find (*a*) I, in amperes; (*b*) the induction, B; (*c*) the flux θ.

771. (*a*) Find (conditions as in problem 770) the reluctance, $\mathcal{R}$. (*b*) Find by two methods the m.m.f.

772. Find the value for E (due to change of flux, problem 770) in (*a*) c.g.s. units; (*b*) volts.

773. Find the coefficient of self-induction of the coil (problem 770) by means of formula (*a*) $L = 4\pi\ \mu n^2 lA$; (*b*) $E = Ldi/dt$. Give results in henrys.

774. A solenoid of c.s.a. 4 cm^2, having a total of 500 turns of wire, and bent in the form of a ring of mean radius 10 cm, carries a current of 2 amp. (*a*) Find the intensity of the magnetic field H within the coil; (*b*) the magnetic flux; (*c*) reluctance; (*d*) magnetomotive force.

775. Suppose that the solenoid of problem 774 have an iron core of permeability 500 for the given conditions. Find (*a*) H; (*b*) B; (*c*) θ; (*d*) m.m.f.

776. Suppose that the magnetic flux in the coil of problem 774 be reduced to zero in 0.01 sec. What is the average value of the induced e.m.f. in (*a*) c.g.s. units; (*b*) volts?

777. If the current of problem 774 increase from 2 to 12 amp. in 0.1 sec. and the resultant E be 0.2 volt, what is the coefficient of self-induction in henrys?

778. (*a*) Compute the inductance in henrys of a coil a meter long and 50 cm^2 in cross-sectional area, if it has 10,000 turns. (*b*) If the current in this coil increases from 0 to 5 amp. in 0.01 sec., find the average e.m.f. induced.

779. If 110 volts be applied to the coil of problem 778, at what rate will the current increase at the instant the circuit closed? Why will the rate of increase be less immediately afterward?

780. A given solenoid having $25/\pi$ turns per cm, and a c.s.a. of 5 cm^2 is bent in the form of a ring of radius 5 cm. Assuming that the permeability is one, and that the solenoid carries a current of 5 amp., find (*a*) the intensity of the magnetic field (H) within the coil; (*b*) the flux; (*c*) the reluctance.

781. Find (*a*) the magnetomotive force (problem 780) in gilberts; (*b*) the ampere-turns.

782. Suppose that the current in the coil (problem 780) increases from 2 to 12 amp. in 0.1. Find the average e.m.f. induced in volts, μ being 250.

783. Suppose that the solenoid (problem 780) contains a core of steel (curve A, Fig. 93). Find the magnetic flux in this core, the current in the solenoid being 4 amp.

784. Find the reluctance of the system under conditions of problem 783.

785. Find the coefficient of self-induction under the conditions of problem 783.

786. Consider a current to flow around a coil of wire in a clockwise sense. Illustrate by a drawing the direction and sense of the induced magnetic field within the coil (*a*) while the current is rising to its maximum; (*b*) while the current remains stationary; (*c*) while it falls to zero. Illustrate by a drawing the direction and sense of the induced e.m.f. in the coil for the conditions (*a*), (*b*), and (*c*).

787. Assume that the lines of magnetic induction are coming out of the blackboard toward the observer. Consider that a conductor is moving vertically through the field. Draw a line on the board to represent this conductor. Indicate by an arrow the sense of the induced e.m.f. when the conductor moves (*a*) down; (*b*) up; (*c*) to the right.

788. Draw a vertical line in the field and indicate the sense of the induced e.m.f. when the conductor (line) moves (*a*) to the right; (*b*) left.

789. A conductor cuts across a uniform field of area 1 by 1 m, having an induction (B) of 500 lines, in 0.1 sec. Find the average induced e.m.f. in (*a*) c.g.s. units; (*b*) volts.

790. A steel ring having a mean radius of 8 cm and sectional area of 12 cm^2 is wound with 60 turns of wire. If the permeability is 1,500, find the total magnetic flux in the steel when a current of 3 amp. flows in the wire.

791. If the iron ring whose magnetization curve is B, Fig. 93, has a mean radius of 8 cm and a sectional area of 10 cm^2 and is wound with 600 turns of wire, find approximately the current in the wire which will produce a flux of 130,000 lines of induction in the iron.

792. Find the coefficient of self-induction of the coil of the preceding problem when the flux is 130,000.

793. Find the coefficient of self-induction of the same coil (problem 791) when $H = 20$. What change in L would a further decrease in H to 10 gausses produce?

794. If the current in the coil of the preceding problem is increasing at the rate of 25 amp. per sec., find the counter e.m.f. due to self-induction at the instant the flux reaches the value 130,000.

795. Find the energy stored in the magnetic field of the ring under the conditions stated in problem 794.

796. A coil of wire containing 2,000 turns is 1 m long and has a mean cross-sectional area of 40 cm^2. This is sufficiently long so that end effects may be ignored. A current of 5 amp. flows through the wire. Find the intensity of field within the helix.

797. In the preceding problem find (*a*) the magnetomotive force; (*b*) the magnetic flux.

798. If the coil of problem 796 were bent into the form of a ring solenoid and were wound upon an iron ring of mean length 100 cm, and 40 cm^2 of cross-section, permeability 1,200, what would be the magnetic flux for a current of 5 amp.?

799. How would the result of the preceding problem be changed if we imagine an air gap in the ring 1 cm across?

800. The mean radius of a Faraday transformer ring is 10 cm; sectional area 10 cm^2; permeability under given conditions 2,000; number of turns in the primary per unit length two; total number turns in the secondary 50; current at a given instant in the primary 5 amp. Find the mutual inductance M in (*a*) c.g.s. units; (*b*) henrys.

DYNAMO ELECTRIC MACHINES

157. The Dynamo.—When a dynamo is used to transform mechanical energy into electrical energy it is called a *generator;* when it is used to transform electrical energy into mechanical energy it is called a *motor.*

Let us consider first the case of the simple dynamo, as outlined in Fig. 101. The armature ac which is here shown consists of a single coil of wire which rotates in the magnetic field $N.S.$ The component of the velocity of a given conductor *at right angles to the field is* $v \sin \alpha$, where v is the linear velocity of the conductor, and α is the angle which the face of the armature loop makes with a line at right angles to the direction of the field, Fig. 102. Then the

$$\textit{Instantaneous e.m.f.} = e = NBl\, v \sin \alpha$$

where N = total number of conductors cutting the field; B = magnetic induction; l = length of the conductors cutting across the field; $v \sin \alpha$ = component of the linear velocity of the conductors at right angles to the field.

If we consider the average e.m.f. developed during any number of revolutions of the armature, we have

$$\textit{Average e.m.f.} = E = N\phi/t \textit{ c.g.s. units} = N\phi/(t \times 10^8) \text{ volts},$$

in which N = number of conductors in the armature, or coil; ϕ/t = total number of lines of induction cut per second, or the change of flux per second. It is important to note that for each revolution of the armature, each conductor cuts all the lines of the field *twice.*

Example.—Consider a coil consisting of a single loop moving in a circular path through a uniform field, as shown in Fig. 102. The induction of the field is 50 lines per cm². The length of one conductor (conductor a, Fig. 101, for example) is 30 cm. The coil makes 3,600 r.p.m. The radius of the circular path is 10 cm. Find the instantaneous value of the e.m.f. when the coil has advanced 30° from the vertical position shown in Fig. 102.

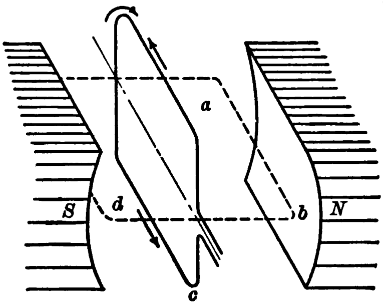


FIG. 101.—Ideal simple dynamo.

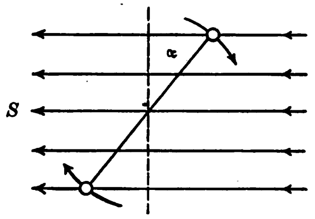


FIG. 102.—Rotation of armature in field.

Solution.—Since there are two wires cutting the field at every instant, we may write $e = NBl\,v \sin \alpha = 2 \times 50 \times 30 \times 2\pi 10 \times 60 \times \sin 30° = 1{,}800{,}000\pi$ *c.g.s. units* $= 0.018\pi$ *volts.*

Example.—An armature consisting of a single loop, Fig. 101, makes 120 r.p.m. The flux through the armature coil, when its face is at right angles to the direction of the field, is 20,000. Find the average e.m.f. induced in volts.

Solution.—In this case $N = 2$. Also, since each wire cuts the field twice during each revolution of the armature, $\phi = 2 \times 20{,}000$. The time of one revolution is $t = 60/120 = 0.5$ sec. Then $E = (2 \times 2 \times 20{,}000)/(0.5 \times 10^8) = 0.0016$ volt.

158. The Fundamental Equation of the Generator.—For purposes of illustrating and explaining the fundamental equation of the generator, we shall consider a two-pole machine of the ring armature type, Fig. 103. The armature rotates in a clockwise sense. By means of the right-hand rule we may show that the e.m.f. at *a* is directed toward the brush *A*; and in *b*, the e.m.f. is directed away from brush *B*. Then *A′* is the positive terminal of the machine and *B′* is the negative terminal.

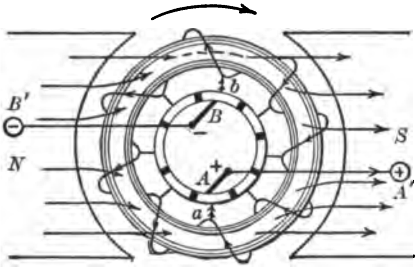


FIG. 103.—Two-pole, two-path dynamo.

It will be noted that in a dynamo of the type shown in Fig. 103 there are *two* parallel paths through the armature from terminal to terminal (*A′* to *B′*). In a four-pole dynamo of the corresponding type, Fig. 104, there are *four* parallel paths from terminal to terminal.

The equation which expresses the relation between the e.m.f. of the dynamo on one hand, and the flux, the number of poles, the number of paths in parallel between the terminals, and the speed of the armature on the other is called the fundamental equation of the generator, and it is written

$$E = pN\phi n/p' \text{ c.g.s. units} = pN\phi n/(p' \times 10^8) \text{ volts,}$$

where E = the e.m.f. developed in the armature; p = number of poles; N = the number of conductors; ϕ = flux per pole; n = number of revolutions per second; p' = number of electrical paths in parallel between the terminals.

Example.—In a given four-pole dynamo the flux per pole is 250,000 lines; the number of conductors on the outside of the armature is 200; the speed is 1,200 revolutions per minute. Find the e.m.f.

Solution.—$E = pN\phi n/(p' \times 10^8) = (4 \times 200 \times 250{,}000 \times 20)/(4 \times 10^8) = 10$ volts.

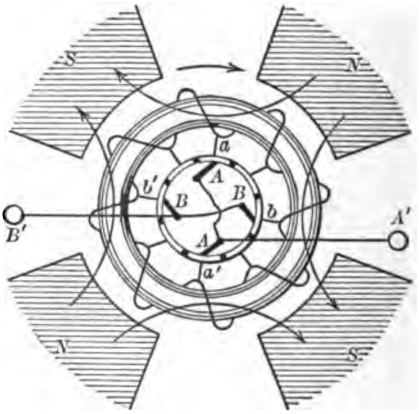


FIG. 104.—Four-pole, four-path dynamo.

159. The Transformer.—Neglecting small losses due to hysteresis and eddy currents in the transformer, we may write $EI = E'I'$, in which EI and $E'I'$ are the e.m.f. and currents in the primary and secondary respectively.

It may also be shown that $E/E' = N/N'$ where N is the number of turns in the primary, and N' is the number in the secondary.

160. The Fundamental Equation of the Motor.—The e.m.f. applied to a motor is used in overcoming the counter e.m.f. developed, and in forcing a current through the armature and field. The counter or induced e.m.f. due to the rotation of the armature in the magnetic field is $E' = pN\phi n/p'$, where p is the number of poles, N the number of conductors on the outside of the armature, ϕ the flux per pole, n the number of revolutions per second, and p' the number of parallel paths from the terminal to terminal. The fundamental equation of the motor is

$$E = E' + I_aR_a = (pN\phi n/p') + I_aR_a,$$

where E = applied e.m.f.; E' = counter e.m.f.; I_aR_a = fall of potential over the armature.

NOTE.—In all problems relating to the motor it will be understood that E' = the induced or counter e.m.f., and E = the applied e.m.f.

161. Work Done by a Motor.—The work done by a motor is a function of the counter e.m.f. developed in the armature, the current flowing through the armature, and the time; that is

$$W = E'I_at.$$

When E', I_a, and t are expressed in c.g.s. units W is in ergs; when E' is in volts, I_a in amperes, and t in seconds, W is in joules.

162. Efficiency of the Dynamo.—The efficiency of a dynamo (generator or motor) may be expressed as the ratio of the power output to the power input; that is

$$\textit{Efficiency} = \textit{output}/\textit{input},$$

in which *output* = *input* − *losses;* or, on the other hand, *input* = *output* + *losses.* The losses in a dynamo are, in general, of two kinds: (*a*) copper losses, and (*b*) stray power losses.

Copper losses = $I_a^2R_a$ heat losses in the armature + $I_f^2R_f$ heat losses in the field.

Stray power losses = S = hysteresis and eddy-current losses + friction losses.

The efficiency of a dynamo may be expressed in terms of (*a*) the commercial or true efficiency, or as (*b*) the partial efficiency. The commercial efficiency is the ratio of the output to the input, taking into account all the losses. In computing the partial efficiency, on the other hand, some of the losses are neglected. There are several types of partial efficiency. In the problems of this text, we shall consider only that partial efficiency of a generator known as the electrical efficiency; and in the case of the motor that partial efficiency known as the efficiency of conversion, or the armature efficiency.

It should be noted also that in the following problems we shall consider only the case of simple shunt-wound and series-wound machines, and not that of the compound-wound dynamo.

163. Generator Efficiency.—We shall consider (*a*) the commercial or true efficiency of the generator, and (*b*) the electrical efficiency.

Commercial efficiency = output/(output + copper losses + stray power losses) = $EI/(EI + I_a^2R_a + I_f^2 R_f + S)$.

Electrical efficiency = $EI/(EI + I_a^2R_a + I_f^2R_f)$.

Example.—The output of a shunt-wound generator is 50 amp. at 110 volts. The stray power loss (S) is 400 watts. The resistance of the armature is 0.2 ohm; that of the field, 55 ohms. The current in the armature is 52 amp.; the current in the field is 2 amp. Find (*a*) the commercial efficiency; (*b*) the electrical efficiency.

Solution.—(*a*) $C.E. = EI/(EI + I_a^2R_a + I_f^2R_f + S) = (110 \times 50)/(110 \times 50 + 52^2 \times 0.2 + 2^2 \times 55 + 400) = 82.5$ per cent. (*b*) $E.E. = 110 \times 50/(110 \times 50 + 52^2 \times 0.2 + 2^2 \times 55) = 87.8$ per cent.

164. Motor Efficiency.—In the case of the motor, we shall consider (*a*) the true or commercial efficiency, and (*b*) the efficiency of conversion or armature efficiency.

Commercial efficiency = (input − copper losses − stray power losses)/input = $(EI - I_a^2R_a - I_f^2R_f - S)/EI$.

Armature efficiency = (counter e.m.f. developed in the armature × armature current)/(applied e.m.f. × total current furnished to the motor) = $E'I_a/EI$.

Example.—A shunt motor running at full-load speed takes 62 amp. at 110 volts. The stray power loss is 400 watts. The armature resistance is 0.02 ohm, and the field resistance is 55 ohms. Find (*a*) the commercial efficiency; (*b*) the efficiency of conversion.

Solution.—The field current $I_f = 110/55 = 2$ *amp.; the armature current therefore* $= 62 - 2 = 60$ *amp.* (*a*) $C.E. = (EI - I_a^2R_a - I_f^2R_f - S)/EI = (110 \times 62 - 60^2 \times 0.02 - 2^2 \times 55 - 400)/(110 \times 62) = 89.8$ *per cent.* (*b*) *The counter e.m.f.* $E' = E - I_aR_a = 110 - 60 \times 0.02 = 108.8$ *volts.* $A.E. = E'I_a/EI = 108.8 \times 60/110 \times 62 = 95.7$ *per cent.*

165. The Force Acting on a Simple Armature Coil.—Let us consider a section of the motor, showing a single armature coil AB, Fig. 105. The current in the armature coil flows in at A, and out at B, thus distorting the magnetic field NS. The reaction of the field against the coil tends to cause the latter to rotate in a counter-clockwise sense. The force acting on each wire is

$$F = BI_al \sin \alpha$$

where F = force in dynes; B = induction; I_a = current in the armature, c.g.s. units; l = length of the wire cutting across the field; and α = angle of rotation, measured from the CD position. The force F is manifestly a maximum when the coil lies in a plane parallel to the direction of the field; that is, when $\alpha = 90°$.

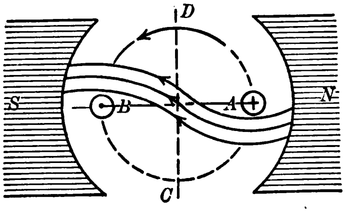


Fig. 105.—Sectional view of motor.

166. Torque.—An expression for the torque developed by a motor may be derived in more than one way. First, for example, if we consider the radius of the armature coil, Fig. 105, to be equal to r, and since torque = Fr, we may write at once

$$Torque = \mathfrak{T} = BI_alr \sin \alpha$$

Second, we may derive an equation involving $\mathfrak{T}$ from the equation for power. Since power $= W/t$, and the work per revolution of the armature $=$ $W = FS = F2\pi r$, we may write, power $= E'I_a = 2\pi Fr/T = 2\pi\mathfrak{T}n$, where n is the number of revolutions per second, and therefore

$$\mathfrak{T} = E'I_a/2\pi n\,.$$

When E' and I_a are expressed in c.g.s. units, and n is the number of revolutions per second, the torque $\mathfrak{T}$ is expressed in dyne-centimeters.

Example.—Given a two-pole two-path motor, having 100 conductors on the armature, a pole flux of 5,000,000 lines, an armature resistance of 2 ohms, a field resistance of 55 ohms, an applied e.m.f. of 110 volts, and a speed of 1,200 r.p.m., to find (*a*) the counter e.m.f. developed; (*b*) the work done by the motor in 2 hr.; (*c*) the mechanical power developed in kilowatts; (*d*) the torque developed.

Solution.—(*a*) $E' = pN\phi n/p'10^8 = 2 \times 100 \times 5{,}000{,}000 \times 20/2 \times 10^8 = 100$ *volts.* (*b*) $I_a = (E - E')/R_a = (110 - 100)/2 = 5$ *amp.;* $W = E'I_a t = 100 \times 5 \times 10^7 \times 2 \times 60 \times 60 = 36 \times 10^{12}$ *ergs* $= 36 \times 10^5$ *joules.* (*c*) $P = E'I_a = 500$ *watts* $= 0.5$ *kw.* (*d*) $\mathfrak{T} = E'I_a/2\pi n = 500/2\pi\ 20 = 12.5/\pi$ dyne-cm.

Problems

801. A rectangular loop of five turns of wire, 20 by 20 cm, lies in a vertical position with its face at right angles to the direction of a magnetic field, the induction of which is 2,000 lines per cm². The armature (rectangular loop) rotates about a median axis, lying in its face, 360 times per min. Find the instantaneous value of the e.m.f. (*a*) when the loop is 30° from the vertical position; (*b*) 60°; (*c*) 90°.

802. Find the average e.m.f. developed per revolution (loop of problem 801) (*a*) in c.g.s. units; (*b*) volts.

803. The loop of problem 802 rotates from a vertical to a horizontal position in 0.1 sec. Find the average current in the coil, assuming that the resistance is 4 ohms.

804. A square frame 10 cm on a side is wound with 10 turns of wire. It rotates about an axis perpendicular to a field of 5,000 lines of induction per cm², 1,200 times a minute. Find the instantaneous e.m.f. produced at the instant the plane of the coil is (*a*) perpendicular to the lines of induction; (*b*) when it makes an angle of 45° with the direction of the field; (*c*) when it is parallel to the lines of induction.

805. A coil of 500 turns of wire, wound on a square frame 40 cm on a side, is rotated about a horizontal axis in a horizontal field which is at right angles to the axis. The average e.m.f.

developed is 6.4 volts and the speed is 600 r.p.m. Find the intensity of the field, assuming the permeability to be unity.

806. A coil of 100 turns of wire is wound on a square frame, the mean area enclosed being 900 cm^2. The coil is revolved in a horizontal field of uniform intensity of 80 gausses in air. It makes 600 r.p.m. Find (*a*) the instantaneous e.m.f. when the coil is horizontal; (*b*) the average e.m.f. per revolution.

807. If a circular coil of wire were made to spin about a vertical diameter first at the magnetic equator, second at the magnetic pole of the earth, explain the e.m.fs. that would be developed in each case.

808. Consider a generator of the type shown in Fig. 104, and assume that the armature rotates in clockwise sense. Determine the positive terminal (A' or B') of the dynamo.

809. A four-pole dynamo having a four-path armature, has a flux in the air gap at each pole of 2,250,000 lines of induction. There are 672 conductors on the armature. The speed is 900 r.p.m. Calculate the e.m.f. of the machine.

810. A two-pole generator has pole faces of area 240 cm^2. There are 200 conductors on the armature in the form of two parallel circuits. The speed is 1,200 r.p.m. The e.m.f. developed is 96 volts. Find the induction in the air gap.

811. A four-pole generator has an armature of the Gramme ring type. There are four brushes and four parallel paths for the current in the armature. The total flux from each pole is 2,500,000 lines of induction. There are 600 turns of wire on the ring. At what speed is it running, if it develops 225 volts?

812. An impressed e.m.f. of 110 volts is applied to a series motor, the armature of which has a resistance of 0.4 ohm. (*a*) What is the counter e.m.f. developed in the armature when the motor is taking a current of 5 amp.? (*b*) If the armature be suddenly stopped, what current will flow through it?

813. In the case of a given motor $p/p' = 1; N = 100; \phi = 1{,}200{,}000$. Find the speed with which the armature would have to run in order to develop a counter e m.f. of 108 volts.

814. Given a shunt-wound generator which delivers a current of 50 amp. at 110 volts. The field resistance is 22 ohms; the armature resistance is 0.15 ohm. The stray power loss is 500 watts. Find (*a*) the commercial efficiency; (*b*) the electrical efficiency.

815. Given a shunt motor to which is applied a pressure of

110 volts. The field resistance is 44 ohms; the armature resistance is 0.14 ohm. The current supplied to the motor is 50 amp. The stray power loss is 700 watts. Find (*a*) the true efficiency (commercial); (*b*) the armature efficiency.

816. A two-pole shunt motor, having a two-path armature, has a flux of 2,000,000 lines of induction through the armature. The armature resistance is 1.67 ohms; the field resistance is 55 ohms. The armature current is 40 amp.; the field current is 2 amp. The speed is 900 r.p.m. Find (*a*) the counter e.m.f. developed; (*b*) the electrical work done in 4 hr.; (*c*) the mechanical power developed.

817. Find (*a*) the torque (problem 816); (*b*) the commercial efficiency, the stray power loss being 250 watts.

818. Consider the generators, Figs. 106 and 107, to run at constant speed. If the resistance in the main circuit of each be reduced what will be the effect on (*a*) the field; (*b*) the voltage?

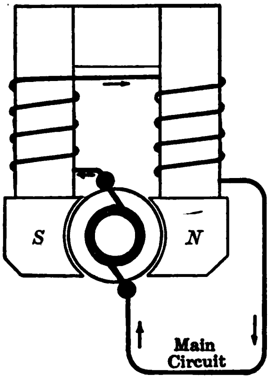


FIG. 106.—Series dynamo.

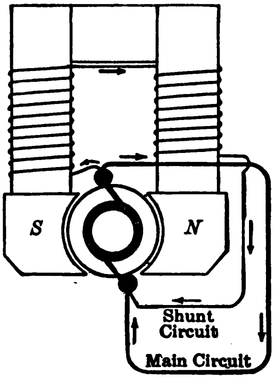


FIG. 107.—Shunt dynamo.

819. A street car motor has a resistance of 1.8 ohms. It is operated on 500 volts applied e.m.f. and takes 20 amp. of current. Compute (*a*) the counter e.m.f.; (*b*) the power edveloped in kilowatts; (*c*) the electrical efficiency. (Street car motors are series machines.)

ELECTROSTATICS

167. Introductory.—It is manifestly not the province of a text of this sort to take into consideration the various theories which seek to explain the nature of electricity, except insofar as such theories may be necessary for the solution of certain practical problems.

We shall agree to call the electricity found on glass when rubbed with silk, positive (+); that found on hard rubber or sealing wax when rubbed with cat's fur or flannel, negative (−). We shall agree, also, to let the condition of the bodies of A and B, Fig. 108, in which the lines represent electrostatic tubes of force, be represented by the simpler outline as shown by A and B in Fig. 109.

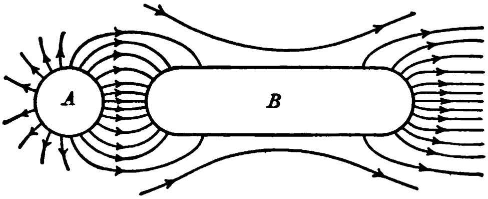


FIG. 108.

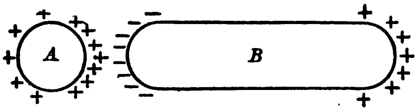


FIG. 109.—Electrified bodies.

The symbols $+Q$ and $-Q$ represent positive and negative charges, respectively. Surface density σ is the charge per unit area; that is $\sigma = Q/A$.

Electric charges of like sign repel; charges of unlike sign, attract. The force of attraction or repulsion as given by Coulomb's law, is

$$F = \pm QQ'/kd^2$$

in which F = force in dynes; Q and Q' = the two charges; d = distance in cm between the points at which the charges may be considered as concentrated; and k = dielectric constant of the medium.

For a vacuum the dielectric constant k is taken as unity. For air k is very nearly unity. For Tables of Dielectric Constants, see page 201.

168. Electrostatic Unit Quantity.—The *unit* of *quantity* is that quantity (or charge) which repels an equal quantity of like sign at a distance of one cm, in a vacuum, with a force of one dyne.

169. Intensity of Electric Field.—The intensity E of an electrostatic field at a given point is measured by the force which is exerted on a unit charge at that point. As in magnetism $H = F/m$, so in electrostatics $E = F/Q$. From Coulomb's law we may then write

$$E = \pm Q/kd^2.$$

The intensity of an electrostatic field E at any point is a vector quantity, and has the same direction and sense as the force acting upon a unit positive charge at the given point.

Example.—Two charges Q and Q' are at a distance of 30 cm apart in air ($k = 1$). The charge Q is +100 units; Q' is −800 units. Find the magnitude, direction, and sense of the electrostatic field at a point p between the two charges, and 10 cm from Q.

Solution.—Consider a unit positive charge to be placed at p. *Then the intensity of the field at* p *with respect to* Q *is* $E = +100/10^2 = +1$, *the* + *sign indicating repulsion. Also, the intensity of the field at* p *with respect to* Q' *is* $E' = -800/20^2 = -2$, *the* − *sign indicating attraction. The sum of these two vectors* = 3 *dynes having the direction and sense* p *to* Q'.

170. Electrostatic Induction.—As in magnetism, electrostatic induction corresponds to a strain in an elastic body, while intensity of field corresponds to a stress. In magnetism the relation of strain to stress is expressed by the equation $B = uH$; in electrostatics we write $D = kE$, where D is the electrostatic induction; E the electric stress; and k the dielectric constant.

If we agree to represent unit induction by one line per square centimeter, then the total number of lines of induction passing through a sphere of unit radius, described about a point charge of Q units as a center, is $4\pi Q$.

171. Electrostatic Potential.—Consider two points p and p' in an electrostatic field due to a charge Q. The difference of potential V between the two points is numerically equal to the work done in moving a unit positive charge from p' to p. It may be shown by methods of the calculus that the potential of any point p with reference to the charge Q is

$$V = Q/kd,$$

in which the sign of Q (+ or −) determines whether the potential V is positive or negative.

Since a charge Q uniformly distributed upon a sphere acts as if it were concentrated at the center, it follows that d, in the equation above, is the distance from p to the center of the sphere upon which Q is uniformly distributed.

FIG. 110.

Example.—The charge Q, Fig. 110, is + 120 units. Find the work in ergs required to move a unit positive charge from p' to p the distance from p' to $p = d' = 5$ cm, and $d = 15$ cm.

Solution.—The potential at $p = V = Q/d = 120/15 = 8$, *and the potential at* $p' = V' = 120/20 = 6$. *The difference of potential between* p *and* $p' = V - V' = 2$ *ergs per unit quantity.*

Example.—To find the potential at a point due to a number of concentrated charges, Q, Q', etc. In this case we get the sum of the potentials at p with reference to Q, Q', etc., *taking each with its proper sign* as illustrated by the following example. Two concentrated charges Q and Q' are 30 cm apart. Find the potential at a point p, 10 cm from Q, and 20 cm from Q', when $Q = +100$ and $Q' = -800$.

Solution.—Let V *be the potential at* p, *due to the charge* Q. $V = +Q/d = +10$. *This means that* 10 *units (ergs) of work would be expended upon a unit charge in moving it from* p *to* Q. *Let* V' *be the potential at* p *due to* Q'.

Then $V' = -Q'/d' = -40$. *This means that* 40 *units of work would be expended by a unit positive charge in moving it from p to* Q'. *The total potential then* $= V + V' = (+10) + (-40) = -30$.

172. Capacity.—The relation of a given charge Q to the capacity and potential of the conductor is

$$Q = CV$$

where C = capacity of the conductor, and V = the potential to which it is raised by the charge Q. The *capacity of a conductor* is numerically equal to the quantity required to raise it to unit potential.

173. Capacity of Conductors.—The capacity of a conductor depends upon its shape, its contiguity to other conductors, and dielectric constant k of the medium concerned. The capacity of the following conductors is in each case expressed in c.g.s. units.

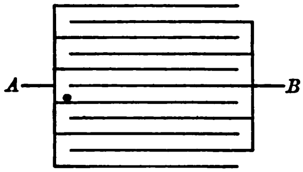


FIG. 111.

Capacity of a sphere = $C = kr$, where k = dielectric constant, and r = radius of sphere.

Concentric spherical condensers = $C = krr'/(r' - r)$, where r and r' are the radii of the two spheres.

Plate condenser = $C = kA/4\pi t$, where A = total area in square centimeters of the dielectric between the conductors; and t = thickness of the dielectric in centimeters.

Cable = $C = kl/2\log_e(r'/r)$, where l = length of cable in centimeters; k = dielectric constant of the sheath; r' = radius of the sheath or envelope; and r = radius of the core.

Single wire = $C = l/2\log_e(l/r)$, where l = length in centimeters; r = radius of wire in centimeters.

Aerial twin wires = $C = kl/4\log_e(d/r)$, where l = length in centimeters; d = distance in centimeters, between the wires; r = radius of each wire.

Example.—A plate condenser consists of eleven sheets of tinfoil, as shown in Fig. 111, the area of each sheet being 400 cm^2. The conductors (sheets of tinfoil) are separated by glass of thickness 5 mm, having a dielectric constant 8. Find the capacity of this condenser in c.g.s. units.

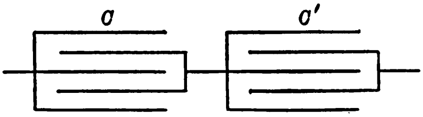


FIG. 112.—Condensers in series.

Solution.—*The eleven sheets of tinfoil enclose ten thicknesses of the dielectric, the area of each being* 400 *cm*^{2}. *From the equation* $C = kA/4\pi t$ *we may write* $C = (8 \times 10 \times 400)/(4 \times \pi \times 0.5) = 16{,}000/\pi$ c.g.s. units.

174. Capacity of Condensers in Series and Parallel.—*Series.*—The capacity of condensers connected in series, Fig. 112, as expressed in reciprocal quantities, is

$$1/C = 1/C' + 1/C'' + \ldots$$

Parallel.—The capacity of condensers in parallel, Fig. 113, is

$$C = C' + C'' + \ldots$$

Example.—Given three condensers, each having a capacity of 6 units, to find the capacity of the system when the three are connected (*a*) in series; (*b*) in parallel.

Solution.—(*a*) $1/C = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$; *then* $C = 2$. (*b*) $C = 6 + 6 + 6 = 18$.

175. Energy Expended in Charging Condensers.—It may be shown that the energy W required to charge a condenser with a quantity Q to a potential V is

$$W = QV/2 = CV^2/2 = Q^2/2C,$$

where W is expressed in ergs when Q, C, and V are expressed in c.g.s. units.

176. Loss of Energy Due to Dividing a Charge.—Let us take as a special case for illustration two spheres A and B, having equal radii r. Sphere A is charged with a quantity Q to a potential V. It is then brought into contact with B by means of a thin wire. A spark appears when the contact is made between the spheres. The energy of the charge on A before contact with B is $W = Q^2/2C = Q^2/2r$. Since the two spheres have equal radii their capacities are equal. Therefore, after contact, Q will be equally distributed over the two spheres, and the energy of each will be $W = (Q/2)^2/2r$. The energy due to the charge on the two spheres is now $W = (Q/2)^2/2r + (Q/2)^2/2r = Q^2/4r$. The loss of energy is therefore $Q^2/2r - Q^2/4r = Q^2/4r$.

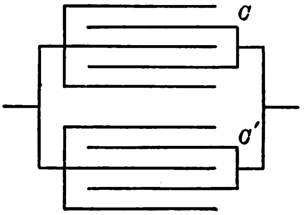


FIG. 113.—Condensers in parallel.

Problems

820. Charges Q and Q' are placed 20 cm apart in air. Charge Q is $+100$ units. Find the force acting between the two charges, when (*a*) Q' is $+100$ units; (*b*) -100 units.

821. Find the field intensity E under conditions named in problem 820 at a point p midway between Q and Q'.

822. Find the potential V at the point p, conditions as in problem 820.

823. A charge of 4,000 electrostatic units is placed upon a sphere which is free from the influence of other charges. The radius of the sphere is 10 cm. Find (*a*) the surface density (σ) of the charge; (*b*) the capacity of the sphere.

824. (*a*) Find the potential of the sphere (problem 823). (*b*) Find the potential at a point 10 cm from the surface of the sphere.

825. What is the energy of the charge on the sphere (problem 823)?

826. Given three condensers, each having a capacity of 300 electrostatic c.g.s. units. Find the capacity of the system when the condensers are connected (*a*) in parallel; (*b*) series.

827. Find the charge on each system (problem 826) when the charging potential is 300 units. What is the charge on each condenser?

828. Find the energy of the charge on each system, under the conditions of problem 827.

829. Consider a straight line on which there are three points *A*, *B*, and *C*, such that the distance from *A* to *B* is 70 cm and *B* to *C* is 30 cm. A charge of +50 c.g.s. units is placed upon *A*. Find the magnitude, direction, and sense of the force between *A* and *B* when (*a*) *B* is charged with +50 units; (*b*) −50 units.

830. A charge of +50 units is placed at *A*, and −20 units at *B*, problem 829. Find the magnitude and sense of the field intensity (E) at a point (*a*) midway between *A* and *B*; (*b*) at *C*.

831. Find the potential (V) at the two points mentioned in problem 830.

832. A sphere whose radius is 10 cm is charged with 1,000 c.g.s. units of electricity. Find (*a*) the capacity of the sphere; (*b*) its potential; (*c*) the energy due to the charge.

833. Suppose that the sphere of problem 832 be made to divide its charge with a similar sphere. Find the energy due to the charge on both spheres. How do you explain the apparent loss of energy?

834. Sphere *A* has a charge of +20 and an equal sphere *B* a charge of −10. The two spheres are brought into contact for a moment and are then separated to a distance of 20 cm. Find the direction and magnitude of the force acting between them.

835. Two equal spheres charged one with +20 and the other with −15 units of electricity are placed at a certain distance apart. They are then brought into contact and afterward placed in their original position. Find the ratio of the forces acting between them before and after contact.

836. A sphere of radius 10 cm charged with 30 units, is made to share its charge with another sphere of radius 5 cm. The two spheres are 85 cm apart from surface to surface. Find (*a*) the ratio of the two charges, (*b*) the force with which the two spheres repel each other, assuming that the charges act as if at the center of the respective spheres.

837. Three small spheres *A*, *B*, *C* are placed at the vertices of an equilateral triangle, 1 m on each side. A has a positive charge of 10 electrostatic units. Find the magnitude and direction of the resultant force acting on A when (a) B and C are each charged with 10 positive units; (b) with 10 negative units; (c) B has a positive charge of 10 units and C a negative charge of 10 units.

838. Consider A (problem 837) to have a positive charge of 10 units, B a positive charge of 15, and C a charge of 5. B and C are connected for an instant by means of a wire of negligible capacity. Find the direction and magnitude of the force acting on A when (a) C is positively charged; (b) negatively charged.

839. Find the capacity of a condenser made of two concentric spheres having radii of 10 and 8 cm respectively, when the space between the spheres is filled (a) with air; (b) oil of dielectric constant 2.

840. A condenser consists of two parallel circular plates. Find its capacity when (a) the radius of each plate is 10 cm, distance apart 1 mm, dielectric air; (b) radius 12 cm, distance apart 2 cm, dielectric sulphur.

841. Two small spheres are charged with +400 and +100 units of electricity, respectively, and placed 100 cm apart. (a) Find the neutral point in the field, *i.e.*, the place where the intensity of electric field is zero. (b) If the 100 units were negative, where would the neutral point be?

842. Two metal spheres one having a diameter six times as great as the other, are connected by a long thin wire and electrified. Compare their electric charges, potentials, surface densities, and energies.

843. From Coulomb's law we may write $Q^2 = kd^2F$. Show that the dimensional formula for electrostatic quantity, $Q = kM^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$.

844. The defining equation for current, measured in e.m.u. is $H = 2\pi I/r$, from which $I = Hr$. Also, $Q = It = Hrt$. Show that the dimensional formula for electromagnetic quantity, $Q = M^{\frac{1}{2}}L^{\frac{1}{2}}$.

845. Show that the ratio of the two formulæ for quantity, as developed in problems 843 and 844, is of the nature of a velocity, v.

177. Units of Quantity, Potential and Capacity.—The practical electrostatic units of Q, V, and C may be defined in terms of the corresponding c.g.s. units.

Quantity.—The *electrostatic c.g.s. unit of quantity* is that quantity which, in a vacuum, will repel an equal and like charge with a force of 1 dyne. The *practical unit of quantity* is the coulomb = 3×10^9 c.g.s. units.

Potential.—The *electrostatic c.g.s. unit of potential* difference exists between two points when one erg of work is required to move unit quantity from one point to the other. The *practical unit of potential* difference is the volt = 1/300 c.g.s. units.

Capacity.—The *electrostatic c.g.s. unit of capacity* is a capacity such that unit quantity will charge the given body to unit potential. The *practical unit of capacity* is the *farad* = 9×10^{11} c.g.s. units. A microfarad = one millionth of a farad = 9×10^5.

178. Relation of Electromagnetic Units to Electrostatic Units.—The relation of the practical units of current, quantity, potential capacity resistance and inductance to the corresponding electromagnetic c.g.s. units and electrostatic c.g.s. units are given in the following table.

Practical units	Electromagnetic c. g. s. units	Electrostatic c. g. s. units
1 ampere	10^{-1}	3×10^9
1 coulomb	10^{-1}	3×10^9
1 volt	10^8	1/300
1 farad	10^{-9}	9×10^{11}
1 microfarad	10^{-15}	9×10^5
1 ohm	10^9	$1/(9 \times 10^{11})$
1 henry	10^9	$1/(9 \times 10^{11})$

Problems

NOTE.—The problems of this set are intended to illustrate the relation between the practical units and the corresponding absolute units of the electromagnetic and the electrostatic systems. In the solution of these problems it is to be understood that, unless stated to the contrary, the dielectric constant $k = 1$; and also that the symbols e.m.u. stand for electromagnetic absolute units, and e.s.u. designate electrostatic absolute units.

846. A spherical conductor having a radius of 10 cm is given a charge of 300 e.s.u. Find the quantity on the sphere in coulombs.

847. Find the capacity of the sphere (problem 846) in (*a*) e.s.u.; (*b*) mfs.

848. Find the density of the charge on the sphere of problem 856.

849. Find the potential of the sphere (problem 846) in (*a*) e.s.u.; (*b*) e.m.u.; (*c*) volts.

850. Find the intensity of the field at a distance of 10 cm from the surface of the sphere of problem 846.

851. Find the potential in volts at a point 10 cm from the center of its sphere of problem 846.

852. Find the energy of the charge (problem 846) in (*a*) ergs; (*b*) joules.

853. Given an insulated metal sphere radius 10 cm, charged to a potential of 9,000 volts. Find the potential of the sphere in (*a*) e.s.u. and (*b*) e.m.u.

854. The capacity of the sphere (problem 853) in (*a*) e.s.u.; (*b*) farads; (*c*) mfs.

855. The quantity on the sphere (problem 853) in (*a*) e.s.u.; (*b*) coulombs.

856. The energy of the charge (problem 853) in joules.

857. The intensity of the field (problem 853) at a distance of 10 cm from the surface of the sphere. Name the unit in which the intensity is expressed.

858. The capacity of a condenser is 10 mfs. Find its capacity in (*a*) electromagnetic c.g.s. units; (*b*) electrostatic c.g.s. units.

859. Three condensers, capacities 2, 3, 5 mfs. respectively, are charged in parallel by means of a 110-volt circuit. Find, in practical units, (*a*) the capacity of the system; (*b*) charge (quantity) on each condenser; (*c*) potential of each condenser.

860. Compute the energy of the system (problem 859).

861. Solve problem 859, assuming the condensers to be connected in series.

862. A conducting sphere of 30-cm radius is charged with a quantity equivalent to 0.001 coulomb. If it divides its charge with another insulated sphere of 10 cm radius, what will be the charge on each sphere in e.s.u., and how will the energy of the system compare with the energy of the charge on first sphere?

863. The coatings of a Leyden jar are 500 cm^2 each; the glass 2 mm thick; the dielectric constant of the glass, 8. Find (*a*) the capacity of the jar; (*b*) the energy in ergs of the charge when the jar is charged to a potential of 30,000 volts.

864. Given a condenser having nine sheets of tin foil. These conductors (sheets of tin foil) are separated by glass 1 mm in thickness, $k = 5$, the area of each dielectric being 20 by 20 cm. Find the capacity in mfs.

865. Given three condensers having capacities of 4, 6, and 8 mfs. respectively. Make drawings to illustrate these three condensers connected (*a*) in series and compute capacity; (*b*) in parallel, and compute capacity.

866. A condenser having a capacity of 10 mfs. is charged by means of a 220-volt circuit. Find, in c.g.s. units, (*a*) the charge; (*b*) the energy.

867. A given condenser contains as dielectric 100 sheets of paraffined paper, the dielectric constant of which is 2.1. Each sheet has thickness of 0.005 cm and an area of 20 by 20 cm. Find the capacity in mfs.

868. It is desired to build up a capacity of 2 mfs. by using three condensers of 4, 1.2, and 1 mfs. capacity respectively. Show how this may be done.

869. It is desired to construct a plate condenser having an area 40 by 40 cm, using tin foil conductors, separated by a substance the dielectric constant of which is 3. The thickness of the dielectric between sheets of tin foil is to be $\frac{1}{3}\pi$ cm. According to the specifications the capacity of the condenser is to be 0.08 mfs. How many square centimeters of tin foil will be required?

870. Given a strip of tin foil 20 cm wide and 120 cm in length. A condenser is made by cutting this tin foil into two equal pieces which are separated by a sheet of paraffin 2 mm in thickness. Find the capacity of the condenser.

871. Suppose that the two strips of tin foil of problem 870 had been cut into squares 20 cm on each side, and made into a condenser, each sheet of tin foil being separated by paraffin 2 mm thick. Find the capacity of this condenser.

872. Suppose that the squares of tin foil of problem 871 had been made into separate condensers, each consisting of two sheets of tin foil separated by a sheet of paraffin 2 mm thick, and the condensers connected in parallel. Find the capacity.

873. Find the capacity of the condenser system (problem 872) when the units of the system are connected in series. Express the result in farads.

874. Show that the capacity of a plate condenser, measured in mfs., is $C = 885kA/(d \times 10^{10})$.

875. Show that the capacity of a cable, measured in mfs., is $C = 2.413kl/10^7 \log_{10} (r'/r)$.

876. The radius of the metallic core of a given cable is 1 cm; that of the insulating sheath is 1.5 cm. The dielectric constant k of the sheath is 2.3. Find the capacity, in mfs., of this cable per mile.

877. Show that the capacity of aerial twin wires, measured in mfs., is $C = 12kl/10^8 \log_{10} (d/r)$.

878. What is the capacity per mile of a line consisting of two wires, 0.1 in. in diameter, and 20 in. apart? Give results in mfs.

879. Find the capacity of a single telegraph wire in air, of radius 2 mm, length 10 miles. Express your result in mfs.

ALTERNATING-CURRENT PHENOMENA

179. Methods of Representing Alternating E.M.F. and Currents.—An harmonic alternating e.m.f. or current is one which obeys the laws of simple harmonic motion. An harmonic E and I may be represented by means of (*a*) a sine curve, (*b*) a clock diagram, or (*c*) an algebraic equation.

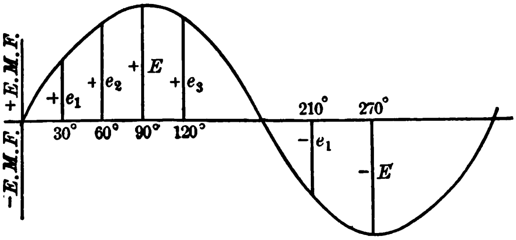


FIG. 114.—Alternating e.m.f. curve.

(*a*) In Fig. 114 we have represented an harmonic alternating e.m.f. The letters e_1, e_2, e_3, etc., represent instantaneous values; $+\mathbf{E}$ and $-\mathbf{E}$ represent maximum values. A similar curve may be used to represent an A.C. The portion of the curve shown in Fig. 114 represents a cycle, in which e_1 is the instantaneous value, at the 30° phase, e_2 the instantaneous value at the 60° phase, **E** is the instantaneous (maximum) value at the 90° phase, and so on to the end of the cycle. The frequency n is the number of times the cycle is completed per second. A 60-cycle system is one in which there occurs 60 cycles per sec; that is, $n = 60$.

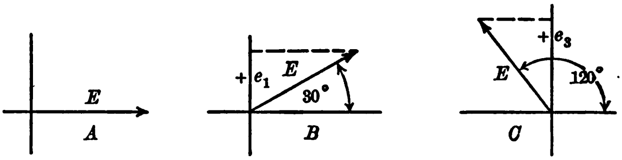


FIG. 115.—Use of clock diagram to illustrate an alternating E or I.

(*b*) The cycle represented in Fig. 114, for example, may also be represented by means of the rotation of a line, numerically equal to **E**, which represents a maximum value, and which rotates in a counter-clockwise sense. Zero phase is shown in Fig. 115 by the position of the vector in *A*; the 30° phase in *B*; the 120° phase in *C*, and so on.

(*c*) The third method of representing an harmonic E and I is by means of the equations

$$e = \mathbf{E} \sin \omega t,$$
$$i = \mathbf{I} \sin \omega t,$$

where e and i = instantaneous values; $\mathbf{E}$ and $\mathbf{I}$ = maximum values; and ωt = the phase angle.

180. Average Values of E and I.—Consider the values of e of the sine curve for a half cycle, as shown in Fig. 114. The quantities e_1, e_2, etc., represent numerical values of the sines of a series of angles, varying from 0° to 90° to 0°.

An average value for e (or i) may be found approximately by getting the mean of a series of values throughout the half cycle. The true average for the half cycle, as determined by methods of the calculus, is

$$AE = 2\mathbf{E}/\pi = 0.636\mathbf{E}$$
$$AI = 2\mathbf{I}/\pi = 0.636\mathbf{I}$$

where AE and AI = average values; and $\mathbf{E}$ and $\mathbf{I}$ = maximum values.

Example.—In a given half cycle representing an alternating e.m.f. curve the following values for e were found: 20, 60, 80, 90, 95, 90, 80, 60, 20, 0, where 95 represents the maximum $\mathbf{E}$. Find (*a*) the approximate average of these quantities by taking the mean of the values given, and (*b*) compare the result with the average value obtained by use of the equation.

Solution.—(*a*) *Approximate* $AE = (20 + 60 + 80 + 90 + 95 + 90 + 80 + 60 + 20 + 0)/10 = 59.5$; (*b*) $AE = 0.636 \times 95 = 60.4$ true value.

181. Effective Values of E and I.—A.C. instruments measure not average values, but square root of average square values. The square root of the mean square value for e and i are called the *effective values*, and are designated by the letters E and I. The effective values (square-root-of-mean-square-values) of E and I, as recorded by A.C. instruments, correspond in magnitude to the E and I values as given by D.C. voltmeters and ammeters. It may be shown that

$$E = 0.707\mathbf{E}$$
$$I = 0.707\mathbf{I}$$

where E and I = effective values; and $\mathbf{E}$ and $\mathbf{I}$ = maximum values.

Example.—Consider the data given in the example of Art. 180. Find the effective values of the e's given (*a*) by taking the square root of the average squares, and (*b*) by use of the equation.

Solution.—(*a*) $(20^2 + 60^2 + 80^2 + 90^2 + 95^2 + 90^2 + 80^2 + 60^2 + 20^2 + 0)/10 = 4{,}602.5$. The square root of this average square $= \sqrt{4{,}602.5} = 67.84$ = approximate value; (*b*) $E = 0.707 \times 95 = 67.17$ = true value.

182. Symbols.—As explained in Arts. 180 and 181, the following symbols are employed to represent harmonic e.m.fs. and currents:

e and i = instantaneous values of e.m.f. and current,
$\mathbf{E}$ and $\mathbf{I}$ = maximum values,
E and I = effective (square-root-of-mean-square) values.

183. E and I in Phase.—When an harmonic alternating e.m.f. is impressed on a system in which there is resistance R only (no inductance L or capacity C), there results an A.C. which is in phase with the impressed electromotive force, Fig. 116.

184. Angle of Lag.—When an harmonic e.m.f. is impressed on a system containing resistance R and inductance L in series, there results an alternating current which *lags behind* the impressed e.m.f. by an angle ϕ, which is called the angle of lag. When we say that a current lags behind an e.m.f. we mean that the current comes to its maximum value at a later time than the e.m.f., as shown in Fig. 117, in which we have represented a current lagging behind the e.m.f. by 30°.

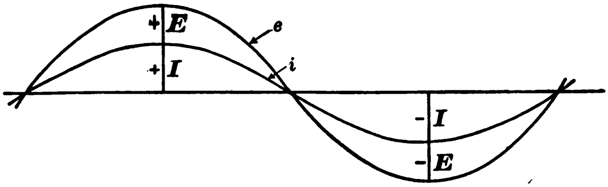


FIG. 116.—Current and e.m.f. in phase.

Starting with the equation $i = \mathrm{I} \sin \omega t$, we get by differentiation an expression for the induced e.m.f., $e = -L di/dt = -L\omega \mathrm{I} \cos \omega t$. Thus we see that i is a sine function of ωt and the induced e is a cosine function of the same angle. This means that i and e differ in phase by 90°. And further, it may be shown from the solution of the differential equation $e = Ri + Ldi/dt$ that $-Ldi/dt$ lags behind the current by a quarter of a period, as shown in Fig. 118. The generator then must furnish an Ri e.m.f. in phase with the current, and it must also furnish an e.m.f. having a value of $+Ldi/dt$ to overcome the $-Ldi/dt$ of self-induction. The impressed e.m.f. is the sum of these two e.m.fs The current thus *lags behind* the impressed e.m.f. by an angle represented by *ab*, Fig. 118.

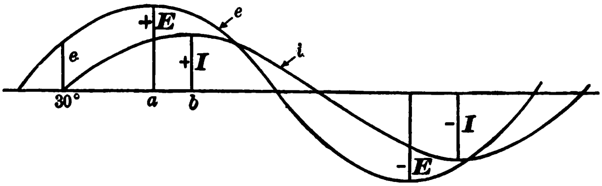


FIG. 117.—Current lagging behind the e.m.f. by 30°.

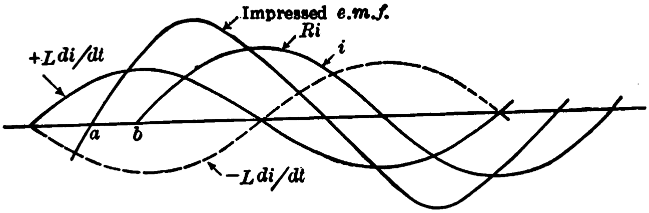


FIG. 118.—Electromotive forces due to R and L in series.

Now for maximum values of the current and the induced electromotive force we have $R\mathrm{I}$ and $+L\omega\mathrm{I}$, where $R\mathrm{I}$ is the component of the impressed e.m.f. in phase with the currrent, and $+L\omega\mathrm{I}$ is the component required to overcome the $-L\omega I$ of self-induction. These two e.m.fs. are at right angles to each other, and therefore may be represented as in the diagram of Fig. 119. The maximum impressed e.m.f. is then $E = I\sqrt{R^2 + L^2\omega^2}$.

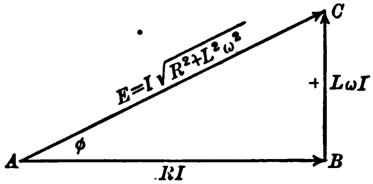


FIG. 119.—E.m.f. diagram.

The angle of log ϕ may be found from the equation:

$$\tan \phi = L\omega/R$$

185. Angle of Lead.—When an harmonic e.m.f. is impressed on a system containing resistance R and capacity C in series, there results an alternating current which *leads* the impressed e.m.f. by an angle ϕ' which is called the angle of lead.

Starting with the equations $e = q/C$, and $i = I \sin \omega t = dq/dt$, and integrating $dq = I \sin \omega t dt$, we have $q = -I \cos \omega t/\omega$. Now the $e = q/C$ of the condenser is in phase with the charge q, and since i and q are sine and cosine functions of ωt respectively, it follows that i and e differ in phase by 90°. It may be shown further that $e = +q/C$, the instantaneous e.m.f. of the condenser, leads the current by a quarter of a period, Fig. 120. Again, as in the case in which we considered the factor of inductance, the generator must supply two e.m.fs.: an Ri e.m.f. in phase with the current, and an $e = -q/C$ to overcome the $+q/C$ of the condenser. The impressed e.m.f., then, is the sum of the Ri and $-q/C$ values. It thus appears that the current *leads* the impressed e.m.f. by an angle represented by *cd*, Fig. 120.

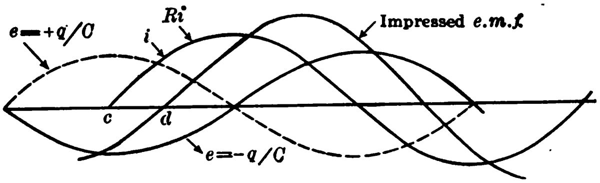


FIG. 120.—Electromotive force due to R and C in series.

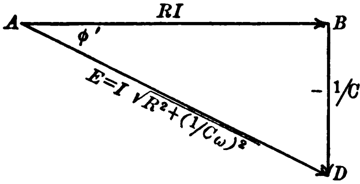


FIG. 121.—E.m.f. diagram.

From the equation $q = -I \cos \omega t/\omega$, we may write for maximum values, $Q = I/\omega$. And since $Q = CE$, we have $E = -I/C\omega$, where E is the maximum e.m.f. furnished by the generator to overcome the $+I/C\omega$ of the condenser. The generator then must furnish two e.m.fs. RI and $-I/C\omega$, which are in quadrature, and which may be represented as in Fig. 121. Here $E = I\sqrt{R^2 + (1/C\omega)^2}$.

The angle of lead ϕ' may be found from the equation

$$\tan \phi' = 1/C\omega R.$$

186. Summary.—In Arts. 184 and 185 we have dealt with maximum values of e.m.f. and current. Since, however, the effective values are equal to a constant times the corresponding maximum values, we may write our equations as follows:

Given R and L in series in an A.C. system, we have

$$\mathbf{E} = \mathbf{I}\sqrt{R^2 + L^2\omega^2},\ \text{or}$$
$$E = I\sqrt{R^2 + L^2\omega^2};$$

and given R and C in series in an A.C. system, we have

$$\mathbf{E} = \mathbf{I}\sqrt{R^2 + (1/C\omega)^2},\ \text{or}$$
$$E = I\sqrt{R^2 + (1/C\omega)^2}$$

When the system contains all three factors, R, L, and C, then the equations become

$$\mathbf{E} = \mathbf{I}\sqrt{R^2 + (L\omega - 1/C\omega)^2},\ \text{or}$$
$$E = I\sqrt{R^2 + (L\omega - 1/C\omega)^2},$$

in which $\mathbf{E}$ and $\mathbf{I}$ = *maximum* values, and E and I = *effective* values.

When $\mathbf{I}$ or I, R, L, C are given in c.g.s. units, $\mathbf{E}$ and E = c.g.s. units; when $\mathbf{I}$ or I is expressed in amperes, R in ohms, L in henrys, C in farads, then $\mathbf{E}$ and E = volts.

187. Reactance and Impedance.—Consider the equation

$$E = I\sqrt{R^2 + (L\omega - 1/C\omega)^2}.$$

The term $L\omega$ is called the *inductive reactance*, and $1/C\omega$ is called the *capacity reactance*. Reactance is measured in ohms. The expression $\sqrt{R^2 + (L\omega - 1/C\omega)^2}$ or $\sqrt{R^2 + L^2\omega^2}$ or $\sqrt{R^2 + (1/C\omega)^2}$ is the *impedance*.

Example.—A 60-cycle A.C. circuit contains in series a resistance of 21 ohms, an inductance of 0.5 henry, and a capacity of 40 mfs. Find (*a*) the inductive reactance; (*b*) the capacity reactance; (*c*) the impedance.

Solution.—(*a*) *Inductive reactance* $= L\omega = 2\pi nL = 6.28 \times 60 \times 0.5 = 188.4$ *ohms.* (*b*) *Capacity reactance* $= 1/C\omega = 1/(2\pi nC) = 1/(6.28 \times 60 \times 0.000040) = 66.3$ *ohms.* (*c*) *Impedance* $= \sqrt{R^2 + (L\omega - 1/C\omega)^2} = \sqrt{441 + 14{,}884} = 123.8$ *ohms.*

Example.—What effective E will be required to maintain an effective current I of 5 amp. in a circuit having a resistance of 5 ohms, and containing a helical coil 1,250 turns, length 25 cm, mean cross-sectional area $10/\pi$ cm², when μ is considered an equal to 20, and the frequency n of the alternating system is 60 cycles per sec.

Solution.—$L = 4\pi\mu n^2Al/10^9 = (4\pi \times 20 \times 2{,}500 \times 10 \times 25)/(\pi \times 10^9) = 0.05$ *henry. The angular velocity* $\omega = 2\pi n = 120\pi$. *Then* $E = 5\sqrt{25 + 355.32} = 97.5$ *volts.*

Example.—A 60-cycle alternating e.m.f. of 110 volts (effective) is applied to a system having a resistance of 7 ohms, and a condenser of capacity C. The current I is 5 amp. Find (*a*) the reactance resistance, and (*b*) the value of C in microfarads.

Solution.—(*a*) $E = 110 = 5\sqrt{7^2 + 1/\omega^2C^2}$. *The reactance resistance* $1/\omega C = 20.85$ *ohms.* (*b*) $\omega = 2\pi n = 120\pi$. *Then* $1/120\pi C = 20.85$. *Hence* $C = 0.000121$ *farads* = 121 *mfs.*

188. Power in an A.C. System.—In a D.C. system the power in watts is equal to the product of volts times amperes; that is, $P = EI$. In an A.C. system, however, this is not necessarily the case, because of the fact that the current I may be in phase or out of phase with E. In the case of the A.C. the power expended at any instant is $p = ei$, in which e is the instantaneous e.m.f. in volts, and i the instantaneous current in amperes. Since e and i may be in phase or out of phase, it follows that a consideration of power values involves two cases, namely, (*a*) when e and i are in phase, Fig. 122, and (*b*) when e and i are out of phase, Fig. 123. In Fig. 122, we have represented a power curve for e and i in phase. The ordinates of the power are obtained for any given point by multiplying the ordinates of e and i for that point. The curve *abcde* is all on the positive side of the time axis.

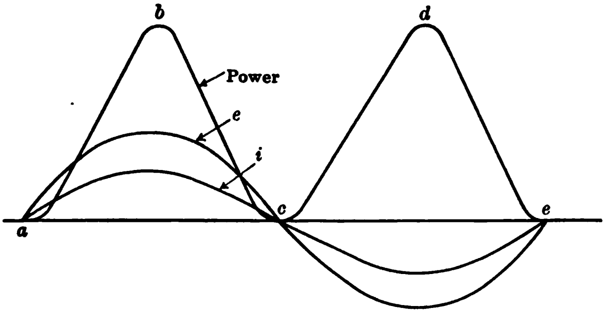


Fig. 122.—Power curves, e and i in phase.

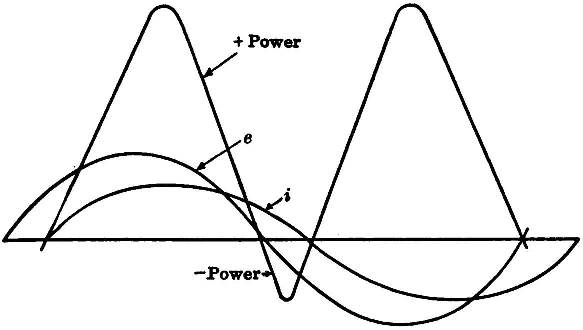


Fig. 123.—Power curve, e and i differing in phase.

In Fig. 123, we have a power curve for an e and i out of phase; that is, i lags behind e by an angle of 30°. In this case some of the ordinates ($e \times i$) are positive and some are negative. This gives part of the curve on the positive side of the axis, and part on the negative side. The positive part of the power curve represents power delivered to the circuit; the negative part of the curve represents power given back from the circuit.

The power delivered by an A.C. system is

$$P = EI \cos \phi$$

when P = power in watts; E = effective volts; I = effective current; ϕ = phase difference between e and i (angle of lag or lead). The quantity $\cos\phi$ is called the *power factor.*

Example.—An alternating e.m.f. of 110 volts is applied to an inductive system in which $R = 60$ ohms, and $L = 0.1$ henry. The frequency is 60 cycles per sec. Find the power expended by the current.

Solution.—$I = E/\sqrt{R^2 + L^2\omega^2} = 110/\sqrt{3{,}600 + 0.01 \times 14{,}400\pi^2}$; hence $I = 1.56$ *amp. To find the angle of lag* ϕ, *we write* $\tan\phi = L\omega/R = 0.6283$, *whence* $\phi = 32.15°$. $\cos\phi = 0.8467$. *Power* $= 110 \times 1.56 \times 0.8467 = 145.3$ *watts.*

Problems

880. Illustrate and explain three ways of representing an harmonic e or i, as follows: (*a*) Sine curve; (*b*) clock diagram; (*c*) equation.

881. Define and illustrate the following: Instantaneous, maximum, average, and effective values of e.m.f. and current.

882. (*a*) Find the instantaneous value of an harmonic e.m.f. at 30°, the maximum value of which is 110 volts. (*b*) Instantaneous value of an harmonic e.m.f. at 45° is 60 volts, find the maximum value. (*c*) Find the maximum value of an A.C. the instantaneous value of which at the 60° phase is 8 amp.

883. (*a*) Find the average value of an A.C. whose maximum value is 70 amp. (*b*) An A.C. at 30° has an instantaneous value of 4 amp. To what D.C. is this A.C. equivalent?

884. Make drawings to illustrate the fundamental relations between an alternating harmonic e and i (*a*) in phase; (*b*) i lagging behind e; (*c*) i leading e.

885. Make drawing to illustrate (*a*) i lagging behind e by 30°; (*b*) lagging behind e by 60°; (*c*) leading e by 90°.

886. Drawing to illustrate (*a*) i leading e by 30°; (*b*) leading e by 60°; (*c*) lagging behind e by 90°.

887. Drawing to illustrate power curve when e and i are (*a*) in phase; (*b*) e and i out of phase.

888. Having given the maximum RI and the induced e.m.f., explain how to find the impressed e.m.f. by the vector method. Write the equation and explain each term.

889. Explain how to find the angle of lag due to (*a*) induction in the circuit. Explain $\tan\theta = L\omega/R$.

Given a coil of 1,000 turns, length 20π cm, cross-sectional area 20 cm^2, and resistance 5 ohms. Upon this coil there is impressed an alternating e.m.f. (effective) of 110 volts, having a frequency of 60 cycles, that is, a period T of $\frac{1}{60}$ sec.

890. Find (*a*) the inductance L of the coil described bottom page 165 in (*a*) c.g.s. units; (*b*) henrys.

891. Find (*a*) the angular velocity; (*b*) the angle of lag; (*c*) the inductive reactance.

892. Find the effective value of the current.

893. Find the maximum value of (*a*) the current; (*b*) the e.m.f.

894. Find the value of the current in its 30° phase; (*b*) 120° phase.

895. When the current is passing through its zero phase, what is the instantaneous value of the impressed e.m.f.?

896. What capacity would have to be put in series with the inductance in order to annul the effect of the latter?

897. Find the power expended upon this coil.

898. Plot a power curve to show the instantaneous values of ei, for the conditions given.

Given a coil of 3,000 turns, 30π cm in length, $50/\pi$ cm^2 in cross-sectional area. The resistance of the coil is 6 ohms. The permeability of the medium within the coil is 10. The coil is connected to a 60-cycle 110-volt circuit.

899. Find the inductance of the coil in (*a*) c.g.s. units; (*b*) in practical units.

900. Find (*a*) the effective value of the current; (*b*) the maximum value; (*c*) the average value.

901. Find the phase difference between e and i.

902. Power expended upon the coil.

Given a 60-cycle 110-volt circuit in which there is connected in series a resistance of 20 ohms and a condenser of 120 mfs. capacity.

903. Find (*a*) the capacity reactance of the system; (*b*) the impedance; (*c*) the current flowing in the system.

904. Find the power conveyed by the current.

905. A non-inductive resistance of 20 ohms, a 120-mf. condenser, and a 6-ohm coil, having an inductance of 0.006 henry, are connected in series with a 60-cycle 110-volt circuit. Find the current in the system.

906. Find the phase difference between the current and the e.m.f., problem 905. Does the angle represent a lag or a lead?

CHAPTER IX

LIGHT

INTENSITY OF ILLUMINATION

189. The Law of Inverse Squares.—Geometrically a light wave may be considered to move outward from its source in the form of a series of concentric spheres. Since the area of a sphere varies as the square of its radius (area of sphere $= 4\pi r^2$), it follows that the area illuminated will vary as the square of the distance from its source, and, on the other hand, the *intensity of illumination* (quantity of light per unit area) will vary inversely as the square of the distance from the source; that is,

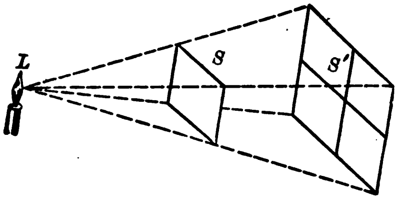


FIG. 124.—Illustrating law of inverse squares.

$$I/I' = d'^2/d^2$$

where I and I' represent respective intensities at the distances d and d' from the source.

The intensity of illumination of a given light may be measured in terms of candlepower (cp.) by means of a photometer. There are, in general, two kinds of candlepower standards: (*a*) the *flame standards* of Great Britain (the Pentane lamp), France (the Bourgie decimale), and Germany (the Hefner lamp); and (*b*) the *electric standards* of the United States. The *American unit of candlepower* is defined in terms of certain tested carbon filament incandescent lamps, kept at the Bureau of Standards, at Washington.

The *international unit of candlepower* = one American electrical unit = one Pentane unit = one Bourgie decimale = 10/9 Hefner unit. The Hefner unit of candlepower is the light given by a horizontal beam from the Hefner lamp, burning pure amyl acetate, at normal atmospheric pressure (76 cm), in an atmosphere containing 8.8 liters of water vapor per cubic meter.

190. Velocity of Light.—The velocity of light was first determined by the Danish astronomer, Roemer, who concluded that light travels with a speed of 186,000 miles per sec. The velocity of light as determined by Michelson is

$$V = 299{,}900 \text{ km/sec.} = 186{,}349 \text{ mi./sec.}$$

Problems

907. Consider Fig. 124: (*a*) How does the quantity of light on the screen in position *S′* compare with *S*? (*b*) How does the intensity (quantity per unit area) on *S′* compare with that of *S*?

908. A candle flame 2 in. in length is placed 6 in. in front of the small aperture in *A*, Fig. 125. Find (*a*) the length of the image on the screen placed 2 ft. from the aperture; (*b*) 5 ft. from the aperture. (*c*) Compare the intensity of illumination of the image for each position of the screen.

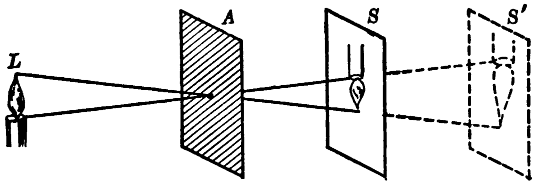


Fig. 125.—Illustrating inverted images.

909. A standard candle placed at a distance of 1 ft. from the screen of a Bunsen photometer gives the same intensity of illumination as that of an incandescent lamp placed at a distance of 4 ft. What is the candlepower of the lamp?

910. If a 2,000-cp. street lamp actually gives 2,000 cp. in a definite direction, at what distance from the lamp will the same amount of illumination be obtained as from a standard candle at a distance of 2 ft.?

911. The nearest fixed star is about 3 light-years from us. Express this distance in miles.

REFLECTION

191. Law of Reflection.—The law of reflection states that the angle of incidence *i* is equal to the angle of reflection *r*, the two angles being in the same plane, Fig. 126.

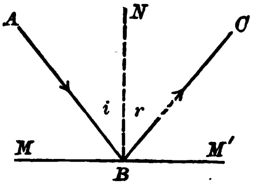


Fig. 126.—Angles of incidence and reflection.

192. Plane Mirrors.—Images formed in plane mirrors are virtual. A *virtual image* is one formed by the apparent focusing of the rays of light from an object, Fig. 127. The virtual image of *A* lies at the point *A′*, on the straight line *AK*, and as far back of the mirror as the object is in front of it.

193. Successive Reflections.—We have given two plane mirrors, set in position such that ϕ represents the angle between their reflecting surfaces, Fig. 128. Suppose that a ray of light from a given source fall upon the mirror M and is reflected twice, once at M and again at M'. Since $\theta = 180 - 2(e + i)$, and $\phi = 90 - (e + i)$ we have

$$\textit{Total deviation} = \theta = 2\phi,$$

that is, for the case of two reflections, the total deviation is twice the angle included between the mirrors. This equation has an important application in the use of the sextant, an instrument for measuring the angle subtended by two distant points (p and p'), or the angular elevation of a point above the horizon.

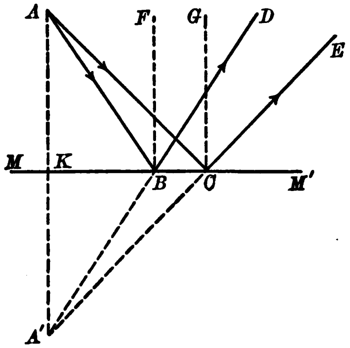


FIG. 127.—Image in plane mirror.

194. Images of Images.—When light is reflected successively from two plane mirrors, the image in the first mirror becomes the object in the second, and so on. If p, Fig. 129, be a luminous point between the mirrors AB and AC it may be shown geometrically that the images p_1, p_2, p_3, etc., lie on a circle, the radius of which is AP. Starting with the object p, images will appear on the circle successively until the arc DE is reached, after which no further reflections will occur. The arc DE lies behind both mirrors.

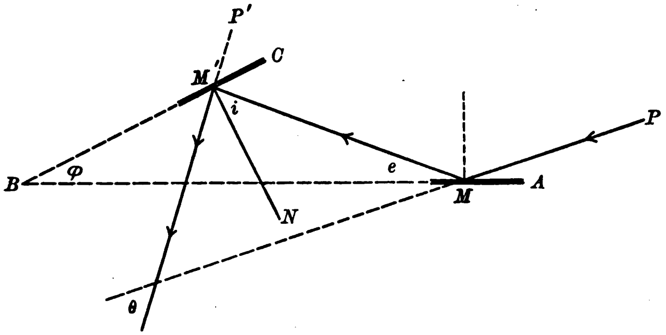


FIG. 128.—Illustrating principles of the sextant.

Let the angle BAC between the mirrors be ϕ, then $2\pi/\phi = n$. When n is a whole number and is even, then the images of the arc DE (p_5 and p_6),

Fig. 129, coincide and the number of images = $n - 1$; when n is a whole number and is odd, the images in *DE* do not coincide, and the number of images = n.

Problems

912. Prove geometrically that the virtual image A', Fig. 127, lies on the straight line AK, and as far back of the mirror as the object A is in front of it.

913. A beam of light falls upon a plane mirror which lies in a horizontal position on the ground 20 ft. distant from the vertical wall of a building. The reflected ray strikes the wall at a point 10 ft. from the ground. Find the angle of incidence of the light on the mirror.

914. It may be shown geometrically that the virtual image of A, Fig. 127, lies on the straight line AK, and that this virtual image A' appears to be as far from behind the mirror as the object A is in front of it. If the angle of incidence ABF is 40°, and the distance KB is 3 ft., what is the distance of the virtual image A' from the object A?

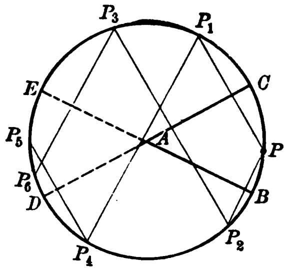


Fig. 129.—Images of images.

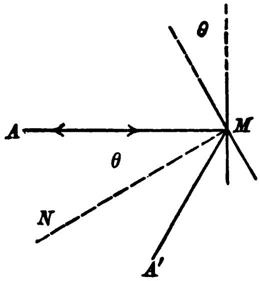


Fig. 130.—Rotating mirror.

915. The distance KB, Fig. 127, is 3 ft.; BC is 10 in.; angle ABK is 50°. Find the angle BAC.

916. Suppose that a ray of light AM, Fig. 130, falls upon a mirror M. The mirror is rotated about M through an angle θ of 5°. Through what arc will the point A be swept, provided AM is 2 ft. in length?

917. Consider Fig. 128. Angle ϕ is 20°; angle $MM'C$ is 36°. Find (*a*) the total deviation due to two reflections; (*b*) the angle AMP.

918. Show by means of a drawing that three images are formed when BAC, Fig. 129, is equal to 90°. Theoretically how many images are possible when the mirrors, AB and AC, are parallel?

919. Find the number of images formed by two plane mirrors placed with two of their edges together, and forming an angle of (*a*) 60°; (*b*) 72°; (*c*) 45°.

195. Spherical Mirrors.—Consider Fig. 131. The opening MM' is the aperture of the mirror. The *vertex* V is a point midway between M and M'.

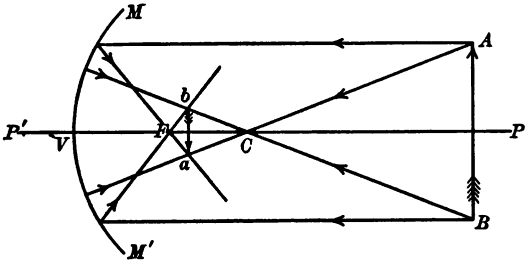


Fig. 131.—Object and image in spherical mirror.

The *center of curvature* C is the center of the sphere of which the mirror is a part. The *principal axis* PP' is a straight line passing through the center of curvature C and the vertex V. AB is an object placed in front of the mirror and ab is its real image. A *real image* is one which is formed by the actual focusing of rays of light. The *object distance*, p is the distance of the object AB from the vertex V; the *image distance*, p' is the distance of the image ab from V.

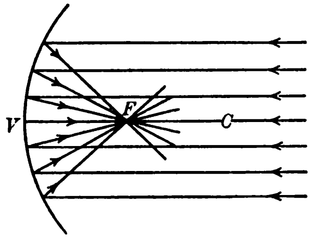


Fig. 132.—Principal focus.

196. Typical Mirror Cases.—There are seven typical cases involving the relation of object and image in spherical mirrors. These seven cases may be illustrated by the following diagrams:

Case I.—Object at an infinite distance, Fig. 132. Image is real, a point, and it lies at the principal focus, midway between V and C.

Case II.—Object at a finite distance, greater than the radius, Fig. 133. Image is real, inverted, smaller than the object, and it lies between F and C.

Case III.—Object at the center of curvature, Fig. 134. Image is real, inverted, same size as object, and it lies upon the object.

Case IV.—Object between F and C, Fig. 135. Image is real, inverted, larger than the object, and it lies beyond the center of curvature C.

Case V.—Object at the principal focus, Fig. 136. Image is real, infinitely large, and it lies at an infinite distance from the mirror.

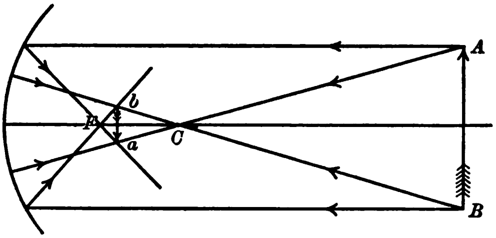


Fig. 133.

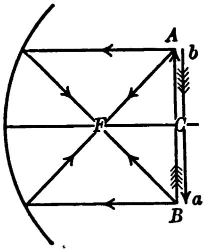


Fig. 134.

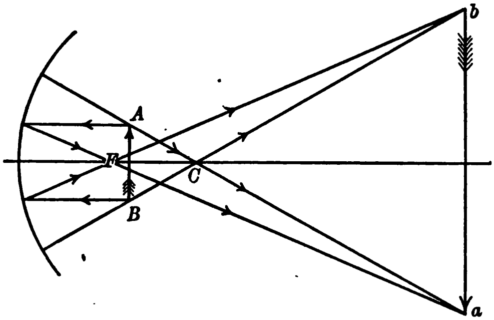


Fig. 135.

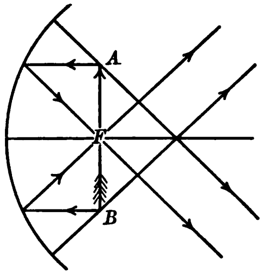


Fig. 136.

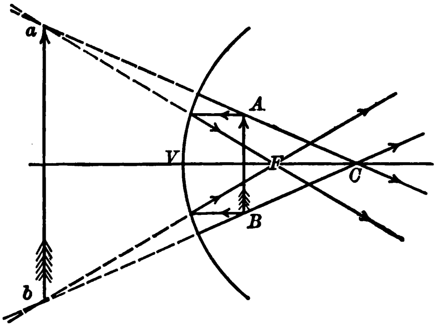


Fig. 137.

Case VI.—Object between V and F, Fig. 137. Image is virtual, erect, larger than the object, and it lies on the *negative* side of the mirror.

Case VII.—Object in front of a convex mirror, Fig. 138. Image is virtual, erect, smaller than the object, and it lies on the *negative* side of the mirror, between F and V.

197. General Mirror Formula.—The general formula for spherical mirrors having a small angular opening is

$$1/p + 1/p' = 2/r = 1/f,$$

where p = object distance; p' = image distance; r = radius of mirror; f = focal length.

When the object is at an infinite distance $p = \infty$, $1/p = 0$, and hence $1/p' = 2/r = 1/f$; that is, $f = r/2$. This means that the *principal focus* of a spherical mirror is at a point on the principal axis midway between V and C.

198. The Sign of the Factors p, p′, r and f.—In dealing with the equation $1/p + 1/p' = 2/r = 1/f$, it is of the utmost importance to be able to determine the sign of the factors p, p', r, and f. *Distances measured on the object side of a spherical mirror are positive* (+); *distances measured on the opposite side are negative* (−). It follows that p is always positive.

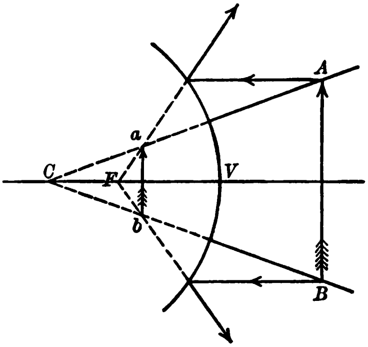


Fig. 138.

In the case of the *concave* mirror, when p is at a distance from V greater than F, Fig. 133, p', r, and f are all on the object side, and hence are positive, and the equation is $(1/+p) + (1/+p') = (2/+r) = (1/+f)$; that is,

$$1/p + 1/p' = 2/r = 1/f.$$

In the case of the *concave* mirror when the object is placed between V and F, the image is virtual, Fig. 137. Here we have $(1/+p) + (1/-p') = (2/+r) = (1/+f)$; that is,

$$1/p - 1/p' = 2/r = 1/f.$$

In the case of a *convex* mirror, Fig. 138, p', r, and f are negative, and the equation becomes $(1/+p) + (1/-p) = (2/-r) = (1/-f)$, or

$$1/p - 1/p' = -2/r = -1/f$$

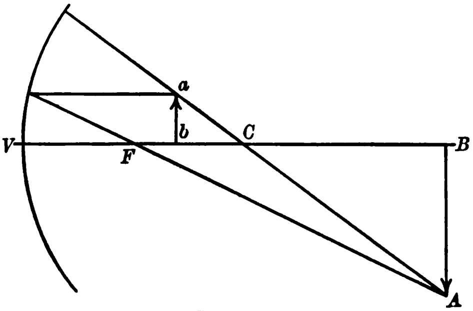


Fig. 139.

199. Size of Object and Image.—A study of the sketches illustrating any one of the cases of reflection, as shown in the preceding article, reveals the fact that the *size of the object is to that of the image as their respective distances from the center of curvature of the mirror.*

Example.—An object 3 in. in length is placed 1 ft. from the vertex of a concave mirror the radius of which is 18 in. What is the size of the image?

Solution.—We first make a sketch illustrating the relative positions of the half object AB and its image ab, Fig. 139. *The triangles ABC* and *abC are* *similar. Hence* $AB : ab = BC : Cb$. *Our next step is to find the value of* BC *and* Cb. *From equation* $1/p + 1/p' = 2/r$ *we write* $1/12 + 1/p' = 2/18$. *Hence* $p' = 36$ *in. Then* $Cb = 18 - 12 = 6$ *in., and* $CB = 36 - 18 = 18$ *in., whence* $3 : x = 6 : 18$ *and* $x = 9$ *in.*

Problems

Make a sketch to illustrate the position and character of the image in the following cases for the spherical mirror; write the formula for each case, and note the signs (+ or −) of the quantities p', r, and f.

920. Object at an infinite distance from the concave side of the mirror.

921. Object at finite distance greater than radius.

922. Object at center of curvature.

923. Object between C and F.

924. Object at F.

925. Object between F and V.

926. Object in front of convex mirror.

927. The radius of a given spherical mirror is 1 ft. An object (candle flame) is placed 24 in. from the mirror on the principal axis, concave side. Find the position of the image.

928. Find the size of the image (problem 927) the length of the candle flame being 3 in.

929. Object placed 9 in. from mirror (problem 927). Find position of image.

930. Find size of image (problem 929).

931. Object placed 3 in. from mirror (problem 927). Find position of image.

932. Find size of image (problem 931).

933. Object placed 1 ft. from mirror (problem 927) on principal axis, convex side. Find position of image.

934. Find size of image (problem 933).

935. Given a spherical mirror of radius 8 in. Find the distance of the image from the vertex of the mirror when an object 4 in. in length is placed 2 ft. from the mirror, on the principal axis, concave side.

936. Object 6 in. from the mirror (problem 935). Find position of image.

937. Find the size of the image (problem 936).

938. Object 3 in. from the mirror (problem 935). Find position of image.

939. Find the size of the image (problem 938).

940. Object 10 in. from the vertex of the mirror (problem 935), on the convex side.

941. Find the size of the image (problem 940).

942. An object is placed between the vertex of a mirror and the principal focus. The image is virtual, erect, and twice the size of the object. The object is 8 in. from the mirror. Find the radius of curvature.

943. A concave mirror has a radius of curvature of 32 in. (*a*) Where must a person stand in front of it in order to see an image of one's face twice its natural size? (*b*) Is the image real or virtual? (*c*) Erect or inverted? (*d*) Make a drawing to illustrate the relation of object to image.

944. Make a drawing to any convenient scale of a concave mirror of 10 cm radius. Place an object 3 cm long, 20 cm from the mirror and find its image. Measure the distance of the image from the mirror and its length and compare these results with those found by computation.

945. What kind of a mirror will produce an erect image of an object one-half of its natural size when the object is 10 in. from the mirror? (*b*) What is the radius of curvature of the mirror?

946. Given two mirrors having the same aperture. Which will make the hotter image of the sun, a concave mirror of 8-in. focal length, or one of 20-in. focal length? Why?

Refraction

200. Defining Terms.—*Refraction* is the bending of a ray of light out of its course due to its passage from one medium to another of different density. Light passing from a rare medium (as air) to a dense medium is refracted toward the normal; light passing from a denser medium to a rare medium is refracted away from the normal.

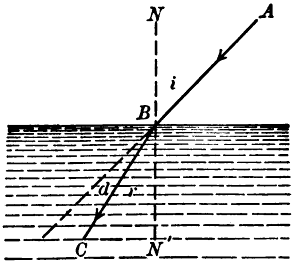


Fig. 140.—Refraction.

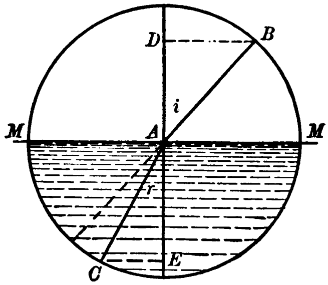


Fig. 141.—Illustrating sin i and sin r.

Consider Fig. 140. Here a ray of light AB is incident at the point B. Angle i = the angle of incidence; r = angle of refraction; d = angle of deviation.

201. Index of Refraction.—Snell's law of refraction states that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant; that is, $\sin i/\sin r = \mu$, in which the constant μ is called the index of refraction.

According to Huygen's principle of refraction, it may be shown that the index of refraction μ with reference to two media is the ratio of the velocity of light in one medium to the velocity of light in the other; or in equational form; $\mu = V/V'$.

Absolute index of refraction is the ratio of $\sin i$ to $\sin r$ when the light passes from a vacuum to the given medium. *Relative index* of refraction is the ratio of $\sin i$ to $\sin r$ when the light passes from one medium to another. The relative index of refraction from air to water may, for example, be designated as μ_{aw}.

If we consider light to pass from the rare medium (as air) to a denser medium (water), Fig. 141, we may write $\sin DAB/\sin CAE = \mu_{aw}$; if, on the other hand, we consider the light to pass from the dense medium to the rare, $\sin CAE/\sin DAB = \mu_{wa}$, from which it follows that $\mu_{wa} = 1/\mu_{aw}$. In general, the index of refraction considered from the dense medium to the rare medium is equal to the reciprocal of the index of refraction from the rare medium to the dense medium.

202. To Trace a Ray of Light from One Medium to Another.—Suppose that we wish to trace a ray from air to water, the angle of incidence being 42° and the relative index of refraction being $\mu_{aw} = 4/3$. Erect a normal at the point of incidence, and determine the angle of refraction from the equation $\sin i/\sin r = \mu$. The angle of refraction in this case is $r = 30° 7'$.

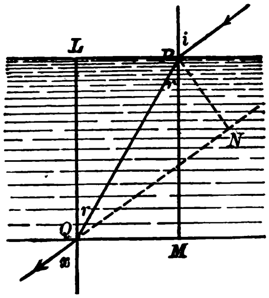


FIG. 142.—Deviation due to refraction.

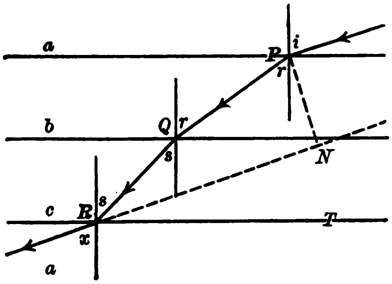


FIG. 143.—Refraction through several media.

203. Refraction through Plane Parallel Plates.—Light refracted through a medium bounded by plane parallel surfaces, Fig. 142, suffers no change in direction, but does undergo a lateral displacement. It may be shown that their lateral displacement PN is

$$PN = t \sin (i - r)/\cos r$$

where PN = lateral displacement; t = thickness of plate; i = angle of incidence; and r = angle of refraction.

204. Refraction through Several Media.—In the case of the refraction of light through several media, as for example, from air to water, to glass, to air, Fig. 143, we may write

$$\mu_{wg} = \mu_{wa}/\mu_{ga} = \mu_{ag}/\mu_{aw}$$

Thus we may say that the relative index of refraction for any two media, as from water to glass (μ_{wg},) may be expressed in terms of the relative indices of the given media to some third medium, as air.

Example.—The relative index of refraction for a given specimen of glass $\mu_{ag} = 3/2$; the relative index from air to water is $\mu_{aw} = 4/3$. Find the relative index of refraction from water to glass.

Solution.—$\mu_{wg} = \mu_{ag}/\mu_{aw} = (3/2)/(4/3) = 9/8 = 1.125$.

205. Critical Angle.—Consider light as passing from the dense medium to the rare medium, Fig. 144. The critical angle $ABN' = \alpha$ is that angle of incidence in the dense medium such that the angle of refraction is 90°. Let α be the critical angle, then $\sin \alpha / \sin 90°$ may be written

$$\sin \alpha = 1/\mu,$$

that is, the sine of the critical angle is equal to the reciprocal of the index of refraction.

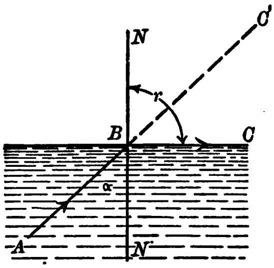


Fig. 144.—Critical angle.

Example.—If an eye immersed in a fluid, the index of refraction of which is 1.42, look out through the horizontal surface, what will be the greatest apparent zenith distance of a star, the light from which just grazes the surface.

Solution.—*Light from the star C, Fig. 144, will be refracted from B to A. The star will consequently appear at C'. The angle NBC' measures the "zenith distance." But* $NBC' = ABN'$ = *the critical angle* α, *and* $\sin \alpha = 1/\mu = 1/(1.42)$. *Hence* $\alpha = 44° \, 46'$.

Problems

947. Taking Michelson's value for the velocity of light in air as 2999×10^7 cm/sec., find the velocity of light (*a*) in water, $\mu = 4/3$; (*b*) in glass, $\mu = 3/2$.

948. A ray of light falls upon crown glass making an angle of incidence equal to 36°. The thickness of the plate is 25 mm; its index of refraction is 1.5. Make a sketch to show the path of the ray through the plate. Find the angle of refraction, and the lateral displacement.

949. A piece of plate glass ($\mu = 1.64$) is placed parallel to the surface of a table. An object on the table viewed through the glass at an angle of incidence of 45° appears to be displaced parallel with the surface of the table by 0.7 cm. Find the thickness of the glass.

950. The refracting angle of a glass prism ($\mu = 1.58$) is 40°. A ray of light falls upon one face of the prism, making an angle of incidence equal to 38°. Trace the ray through the prism, and find the angle which the emergent ray makes with the opposite face.

951. A plate of crown glass is 2 cm thick. A ray of light falls upon this plate, making an angle of incidence of 30°. Find how much the beam is displaced in passing through the plate.

952. A ray of light falls on a piece of plate glass, making an angle of 45° with the normal. The index of refraction of the glass is 1.54. The ray suffers a lateral displacement of 1 cm. Find the thickness of the glass.

953. Consider Fig. 143. Let the medium *a* be air; let the medium *b* be a liquid, the index of refraction 1.6; let the medium *c* be glass, index 1.52. Find the index of refraction from *b* to *c*.

954. Taking the index of refraction of carbon disulphide to be 1.68 find its critical angle.

955. The index of refraction from air to crown glass is 1.512; from air to carbon disulphide, 1.68. Find the index from crown glass to CS_2.

956. Find the critical angle between crown glass and carbon disulphide.

957. Find the velocity of light in a medium whose critical angle is 42°, taking the value for the velocity of light in air as $2{,}999 \times 10^7$ cm/sec.

REFRACTION THROUGH LENSES

206. Defining Terms.—There are two general classes of lenses, convex and concave. A *convex lens* is one that is thicker at the middle than at the edges. A *concave lens* is one that is thinner at the middle than at the edges. Convex lenses may be classified, Fig. 145, as double-convex, plano-convex, concave-convex. Concave lenses, likewise, are divided into double-, plano-, and convex-concave lenses, Fig. 146.

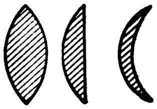


FIG. 145.
Convex lenses.

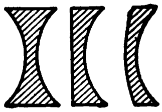


FIG. 146.
Concave lenses.

The line PP', Fig. 147, is the principal axis of the lens; O is the optical center; F is the principal focus. In the case of thin lenses, the focal length f is measured from F to the lens; in the case of thick lenses, the focal length *f* is measured from *F* to the *principal plane* of the lens. In this text we shall deal only with thin lenses, unless specifically stated to the contrary.

207. Typical Lens Cases.—There are five cases involving the relation of object and image in lenses. These five cases may be illustrated by the following diagrams, the first four dealing with the relation of object and image in *convex lenses.*

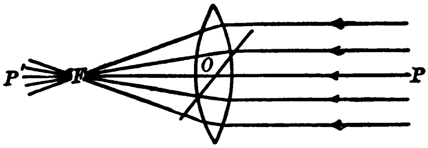


FIG. 147.—Principal focus and optical center of lens.

Case I.—Object at an infinite distance from the lens, Fig. 147. Image is real, a point, and it lies at the principal focus.

Case II.—Object *AB* at a finite distance from the lens, greater than the focal length, Fig. 148. Image *ab* is real, inverted, and it lies on the principal axis beyond the focus *F*.

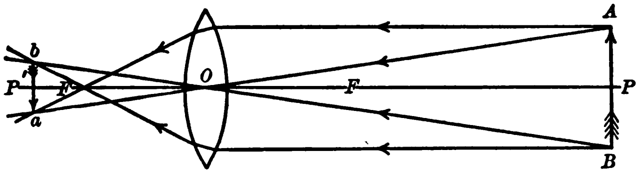


FIG. 148.

Case III.—Object at a distance from the lens equal to the focal length, Fig. 149. Image is real, and at an infinite distance from the lens. This is the conjugate of Case I.

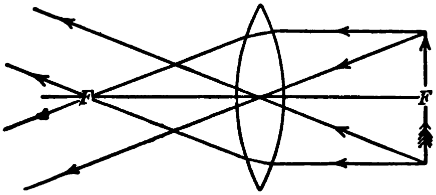


FIG. 149.

Case IV.—Object between the lens and the focus, Fig. 150. Image is virtual, erect, larger than the object and it lies on the object side of the lens. This illustrates the principle of the simple microscope.

Case V.—Concave lens, Fig. 151. Image is virtual, erect, smaller than the object, and it lies on the object side of the lens.

208. The Lens Formula.—For thin lenses it may be demonstrated that

$$1/q - 1/p = (\mu - 1)(1/r - 1/r') = 1/f,$$

in which p = the object distance from the lens; q = image distance; μ = index of refraction of lens; r = radius of curvature of face of lens on the *object side*, and r' = opposite face; f = focal length of the lens.

In using the general lens equation it is of the utmost importance to be able to determine the signs of the various factors entering therein. *Distances measured on the object side of the lens are positive* (+); *distances measured on the opposite side are negative* (−). It follows then that p is always positive; in convex lenses, f is always negative; and in concave lenses f is always positive. For convenience we shall consider the *object side* of the lens as the *right-hand side.*

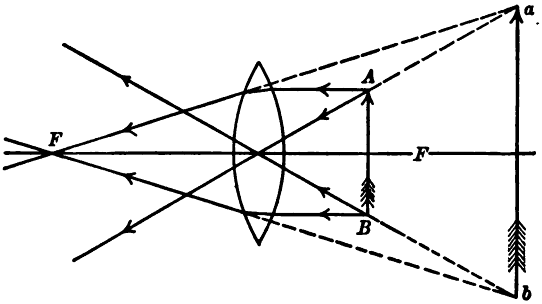


FIG. 150.

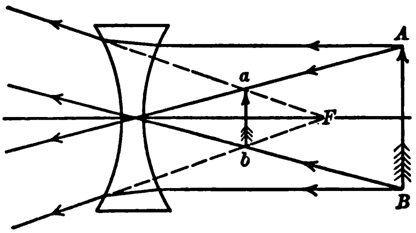


FIG. 151.

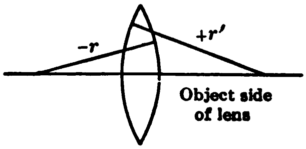


FIG. 152.

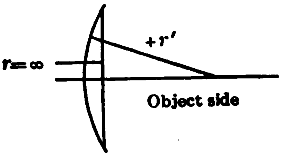


FIG. 153.

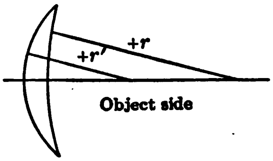


FIG. 154.

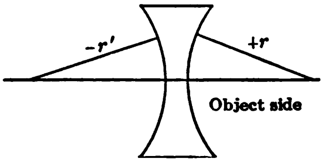


FIG. 155.

The signs of r and r' depend upon the type of lens considered. For example, in Fig. 152 we have $-r$ and $+r'$; in Fig. 153, $+r'$, and $r = \infty$; in Fig. 154, $+r$ and $+r'$; in Fig. 155, $+r$ and $-r'$. It should be noted that in the cases shown in Figs. 153 and 154, the signs of r and r' will be reversed if the lenses are reversed.

Example.—Make sketch of and write the appropriate lens formula for *case II*, Fig. 148.

Solution.—Remembering that the object side of a lens is considered as positive, we may write from Fig. 148, *the following values:* $-q$, $+p$, $-r$, $+r'$, $-f$. *Substituting these values in the general lens equation, we have* $[(1/-q) - (1/+p)] = [\mu - 1][(1/-r) - (1/+r')] = [(1/-f)] = 1/q + 1/p = (\mu - 1)(1/r + 1/r') = 1/f$.

209. Size of Image and Object.—From a consideration of the similar triangles ABO and abO, Fig. 148, or the corresponding triangles of any one of the typical lens cases, we may say that, in general, the size of the image is to that of the object as the image distance is to the object distance; that is $AB : ab = p : q$.

210. Focal Length of a Thick Lens.—A practical method of finding the focal length of a thick lens is that employed in the use of the optical bench, Fig. 156. The lens is first set in position, B, giving a large and distinct image on the screen; it is now placed in position C, giving a small distinct image on the screen. Now $AB = CD$. If we let $AD = l$ and $BC = a$, we may show that

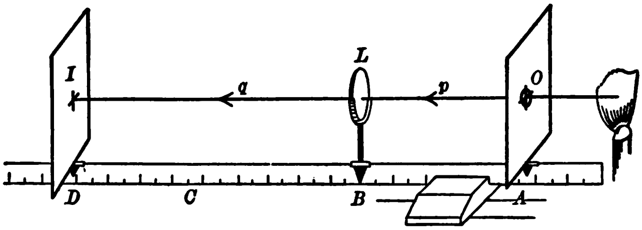


Fig. 156.—Optical bench.

$$f = (l^2 - a^2)/4l$$

Problems

Make sketch to illustrate the position and character of the image in each of the following cases for the double convex lens, and write the appropriate lens formula for each case.

958. Object at an infinite distance from the lens.

959. Object at a finite distance from the lens greater than twice the focal distance.

960. Object at twice the focal distance.

961. Object between twice the focal distance and the focal distance.

962. Object at the focal distance.

963. Object at a point less than the focal distance.

964. Case of the bi-concave lens. Find by drawing the position and character of the image. Write the appropriate lens formula.

965. The focal length of a bi-convex lens is 10 in. Where must an object be placed so that the image will be 16 in. from the lens?

966. An object is placed 60 in. from a bi-convex lens. A real image appears at a distance of 20 in. on the opposite side of the lens. Find the focal length of the lens.

967. The focal length of a bi-convex lens is 1 ft. (*a*) Where must an object be placed so that the image will be 18 in. from the lens and real? (*b*) 18 in. from the lens and virtual?

968. If an object 4 in. in length be placed 3 ft. from a convex lens, the focal length of which is 1 ft., what will be the position of the image? size of the image?

969. If the object (problem 968) be placed 18 in. from the lens what will be the size of the image?

970. The focal length of a double concave lens is 10 in. An object 4 in. in length is placed at a distance of 15 in. from the lens. Find the position and size of the image.

971. How far from a bi-convex lens of focal length 10 in. must a candle flame be placed in order that a distinct image appear on a screen 30 in. distant.

972. Suppose that the candle flame of problem 971, is 3 in. in length and it is placed 2 ft. from the lens, what will be the size of the image?

973. What will be the size of the image (problem 971) if the candle be placed 6 in. from the lens?

974. The focal length of a convex lens is 1 ft. An object 2 in. in length is placed 3 ft. from this lens. Find the position of the image.

975. Where must the object (problem 974), be placed in order that the image be 4 in. in length?

976. If the object (problem 974) be placed 6 in. from the lens, what will be the the size of the image? Will it be real or virtual?

977. The radii of curvature of a flint glass lens ($\mu = 1.6$) are as follows: $r = 10$ cm; $r' = 15$ cm. Determine the signs of r and r', and find the focal length of this lens, when it is (*a*) bi-convex; (*b*) bi-concave.

978. A concave-convex lens has radii as follows: $r = 12$ cm; $r' = 10$ cm. Its index of refraction is 1.5. Find its focal length.

979. Find the focal length of a plano-convex lens, if the radius of the curved surface is 12 cm, and the index of refraction is 1.6. If this lens were to produce an image having the same size as the object, how far would it have to be from the object?

980. A concave-convex lens of crown glass has radii as follows: $r = 12$ cm; $r' = 10$ cm. (*a*) Which face of the lens (concave or convex side) is toward the object? (*b*) Find the focal length.

981. The radius of a bi-convex crown glass lens is 10 in. for each face. Find the focal length for the *D* line (see Table XXX, page 201).

982. A pocket magnifying glass has a focal length of 2 in. Find the size of the image when the object is placed between the lens and the focus and 0.5 in. from the focus.

983. The Washington monument is 550 ft. high. A photograph of it was taken at a distance of a quarter of a mile from the monument. The lens was 6 in. from the plate. Find the length of the picture of the monument.

984. A telescope lens is to be made of crown glass, index of refraction 1.53. The radii are to be equal, and the focal length is to be 15 ft. Determine the radii.

985. The object glass of the Yerkes telescope is 40 in. in diameter; its focal length is 62 ft. Taking the angular diameter of the sun as 32′, and assuming that the image is practically at the focus, find the size of the sun's image produced by this lens. How does the diameter of the lens affect the brilliancy of the image?

986. A convex lens placed at a distance of 25 cm from a candle flame forms a distinct image upon a screen. When the lens is moved 50 cm further from the candle a second image is formed upon the screen. Find (*a*) the focal length of the lens, and (*b*) the distance of the screen from the candle.

987. An object and a screen are 250 cm apart. Where must a lens having a focal length of 40 cm be placed to produce a clear image on the screen. Show that there are two solutions and find the relative size of image and object in each case.

988. A thin double-concave lens, whose radii are 25 cm, is made of glass the index of refraction of which is 1.57? An object 4 cm long is placed 30 cm from the lens. Where is its image, and how long is it?

989. The focal length of a lens is 80 cm. The indices of glass and water are taken as $\frac{3}{2}$ and $\frac{4}{3}$ respectively. Find (*a*) the value of the term $(1/r - 1/r')$; (*b*) the index of refraction of glass with reference to water; (*c*) the focal length of the lens in water.

METHODS OF DETERMINING INDICES OF REFRACTION

211. Spectrometer Method.—The spectrometer is used to determine the indices of refraction of transparent media in the form of prisms. To determine the index of refraction of a liquid by this method it is necessary to place the given liquid in a prismatic vessel having transparent faces. If we let A be the refracting angle of the prism, Fig. 157, and D be the angle of deviation, and if we set the prism for the position of minimum deviation we may write

$$\mu = \sin \tfrac{1}{2}(A + D)/\sin \tfrac{1}{2}A$$

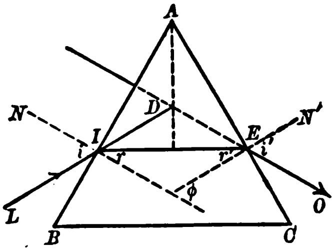


Fig. 157.—Deviation through a prism.

212. Microscope Method.—Let AO be the real depth of a given medium, and let AI be its apparent depth, as viewed with the naked eye, or through a microscope, which is provided with a measuring scale for the determination of the distances AO and AI. The index of refraction in this case is

$$\mu = AO/AI$$

when AO = real depth; AI = apparent depth.

Let us think of two media, A and B, as considered in the above equation. If we let the real depth of B be $AO = d$, and the apparent depth $AI = d'$, then $d' = d/\mu$, where μ is the index of refraction of B with respect to A. In case we consider three media, A, B, C, the real depths of B and C being db and dc respectively, then the apparent depth of $B + C$ is

$$d'' = db/\mu ab + dc/\mu ac;$$

as may be shown by considering the image of the bottom of C as seen from B as an object viewed from A.

213. The Lens Method.—By means of the equation $1/f = (\mu - 1)(1/r - 1/r')$ we may determine μ in terms of f, r, and r'. The focal length f is usually determined by the optical bench method, and the radii r and r' are determined by means of a spherometer.

Problems

990. The refracting angle of a prism is 60°; for the position of minimum deviation, the angle of deviation is 36°. Find the index of refraction of the prism.

991. If the apparent depth of an object below the surface of still water is 3 ft., what is the real depth, μ for water being taken as $4/3$?

992. A fisherman armed with a spear, and standing on the bank of a stream sees a fish lying on the bottom of the stream, which is 3 ft. deep. The fisherman's line of sight makes an angle of 60° with the surface of the water. Where, with reference to the apparent position of the fish, should the fisherman strike in order to hit the fish? Illustrate by sketch.

993. An 18-in. layer of a dense liquid L, which is immiscible in water, and which has an index of refraction of 1.5, is covered with water to the depth of 24 in. Consider the index of refraction of water to be $4/3$. Find (*a*) the apparent distance of the surface of the liquid L below the surface of the water; (*b*) the apparent depth of the liquid L, as observed by an eye immersed in the water.

994. Find the apparent distance from the surface of the water (problem 993) to the bottom of liquid L, as observed by an eye in the air above the water.

995. (*a*) A microscope is focused upon a printed page. A block of glass 12 mm thick is laid over the print and it is now found that the microscope must be raised 4.5 mm in order to produce a clear image. Find the index of refraction of the glass. (*b*) Of what sort of glass would you judge the lens was made?

996. A given lens has a focal length of 10 cm. Its radii of curvature are: $r = 10$ cm; $r' = 15$ cm. Find the index of refraction if (*a*) the lens is bi-concave; (*b*) bi-convex.

997. A concave-convex lens, having a focal length of 1.2 m, has radii of curvature as follows: $r = 12$ cm; $r' = 10$ cm. (*a*) Find the index of refraction of this lens. (*b*) Make a sketch to illustrate which face of the lens is considered as being on the object side.

998. A bi-convex lens ($\mu = 1.6$) has a focal length of 12 cm; $r = 12$ cm. Find r'.

999. The focal length of a certain lens is 12 cm; the index of refraction $= 1.5$; $r = +15$ cm; $r' = -10$ cm. Determine whether the lens is concave or convex.

1000. A 60° prism of flint glass having an index of refraction of 1.6 will produce what angle of minimum deviation?

OPTICAL INSTRUMENTS

214. Dispersion. Fraunhofer Lines.—If a beam of sunlight be passed through a prism, it suffers dispersion, the relative positions of the characteristic colors red, orange, yellow, green, blue, indigo, violet being represented by the Fraunhofer lines, from B to H respectively, Fig. 158.

215. Dispersive Power. Angular Dispersion.—Let μ_B, μ_C, $\mu_D \ldots \mu_H$ be the indices of refraction for the corresponding Fraunhofer lines from B to H. Then *total dispersion* $= \mu_H - \mu_B$; *partial dispersion* $= \mu_C - \mu_B$, $\mu_D - \mu_C$, etc.; *mean dispersion* $= \mu_F - \mu_C$; and *relative dispersion* or *dispersive power* $= (\mu_F - \mu_C)/(\mu_D - 1)$.

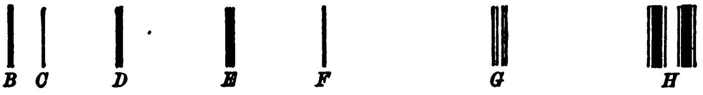


FIG. 158.—Fraunhofer lines.

Consider a prism of small refraction angle, Fig. 159. It may be shown that the angle of deviation for a given line, the H line say, is $D = A(\mu_H - 1)$ and the angle of deviation for any other line, the B line, is $D' = A(\mu_B - 1)$. Then the angular dispersion $= \psi = D - D' = A(\mu_H - \mu_B)$. This equation tells us that for a given refracting medium, the angular dispersion ψ for any two lines may be varied by varying the refracting angle A. It is thus possible for the optician to produce at will prism combinations which will give either deviation without dispersion, or dispersion without deviation.

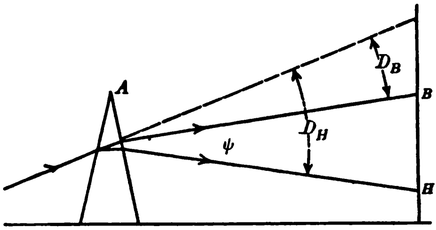


FIG. 159.—Angular dispersion, Ψ.

216. Conditions for Achromatism.—Suppose that we wish to fit two prisms (or lenses) together so as to achromatize certain colors, the B and H lines say. It is necessary to select refracting angles A and A' such that the angular dispersion ψ shall be the same for both refracting media; that is, $\psi = A(\mu_H - \mu_B) = A'(\mu'_H - \mu'_B)$. The condition for achromatism is, then,

$$A/A' = (\mu'_H - \mu'_B)/(\mu_H - \mu_B).$$

217. The Projection Lantern.—The essential parts of the projection lantern, Fig. 160, are (*a*) the source of light; (*b*) condensing lens C, the function of which is to "condense" the divergent rays from the source upon the slide S; (*c*) the focusing or objective lens O. Since it is desired to form on the screen a magnified real image, the object S must be placed at a distance from O greater than the focal length of the objective lens O. The relation of the distance of the object from the lens O to the distance of the screen from O is

$$1/f = 1/q + 1/p$$

in which f = focal length of lens O; p = distance from S to the objective lens O; q = distance from O to screen.

218. The Human Eye.—Mechanically considered, the human eye is a photographic camera, having an automatic focusing system in the muscles that control the crystalline lens, and a sensitive plate (the retina) which reports the image to the brain. The adjustment of the lens system of the eye, such that a distinct image is formed on the retina, is called accommodation.

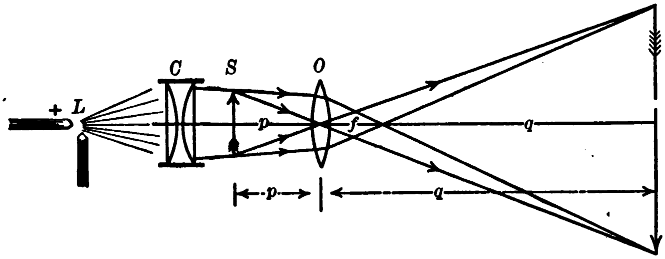


Fig. 160.—Section of projection lantern.

A normal eye can accommodate over all the distances from 6 in. (near point) to infinity (far point). Distance of distinct vision is distance from the eye at which ordinary print can be most easily read. The *distance of distinct vision* for the normal eye = 10 in. = 25 cm.

Example.—If a far-sighted person can see distinctly objects 30 in. away but none nearer, (a) what kind of glasses does he need in order to read at a distance of 10 in.? (b) Determine their focal length.

Solution.—The problem in this case is to find a lens of a focal length such that an object at the distance of distinct vision (10 *in.*) *shall appear to the eye to be at a distance of* 30 *in.; that is* $q = 30$, *and* $p = 10$. *Then* $1/q - 1/p = 1/f$, and $1/30 - 1/10 = 1/f$, *hence* $f = -15$. *The sign of* f *is minus; this means that the lens required is convex.*

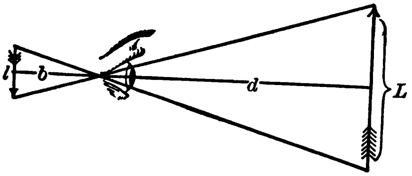


Fig. 161.—Visual angle.

219. Magnification.—The *magnification or magnifying power* of an instrument is the ratio of the apparent size of an object as seen through the instrument under given conditions, to the apparent magnitude as perceived by the eye. Since, however, it is not always convenient or possible to measure the apparent size of an object as seen through the instrument, it is the usual practice to express the magnifying power of an instrument in terms of certain constants of the instrument.

The apparent size of a linear object is measured by the visual angle V which it subtends, Fig. 161. The visual angle may be expressed as

$$V = L/d = l/b = l/L = b/d.$$

220. The Simple Microscope.—In the case of the simple microscope, Fig. 162, the object AB is placed between the principal focus F and the lens, (see Case IV, page 179). The image $A'B'$ is virtual, erect, and larger than the object. Let us consider that the eye is located in such a position that the crystalline lens lies at the principal focus of the lens. We desire to determine the magnifying power of the simple microscope in terms of the visual angle subtended by AB, with and without the use of the lens L. Assuming that $ab = AB$, the visual angle with the lens $= AB/f$; the visual angle without the lens is $AB/25$ cm $= AB/10$ in. in which 25 cm or 10 in. is taken as the distance of distinct vision. Then

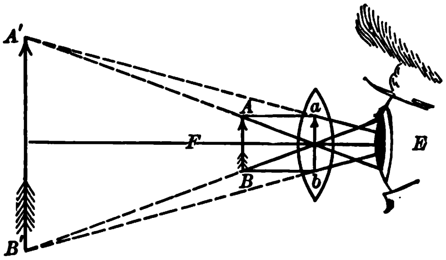


Fig. 162.—Simple microscope.

$$\textit{Magnification} = (AB/f)/(AB/25\text{ cm}) = 25\text{ cm}/f = 10\text{ in.}/f,$$

where f = focal length of the lens.

221. The Compound Microscope.—In Fig. 163 we have shown in outline the relative positions of the two lenses composing the refractive system of a compound microscope. Lens O is the objective; E is the eyepiece. The object AB is placed just outside the principal focus F of the lens O. There are two images; a real image ab and a virtual image $A'B'$.

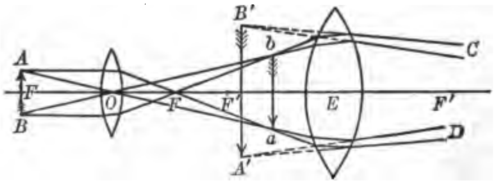


Fig. 163.—Compound microscope.

The magnification (approximate) due to lens O is $(ab/AB) = (L/F)$ where L is the approximate length of the microscopic tube, and F is the focal length of the objective lens O. The magnification due to the lens E is $25\text{ cm}/f = 10\text{ in.}/f$, where f is the focal length of the eyepiece, lens E. The total magnification due to both lenses is

$$\textit{Magnification} = (L/F) \times (25/f) = 25L/Ff.$$

222. The Telescope.—In determining the magnifying power of the astronomical telescope in terms of the focal lengths of the lenses O and E, Fig. 164, it is necessary again to make use of certain approximations. Let the object AB be a star at a great distance from lens O. We assume that the image distance Cp is practically equal to F, the focal length of lens O. The angular measure of the apparent magnitude of the object AB is ab/F. Also, the angular measure of ab, considered with reference to lens E and the eye, is approximately ab/f. The magnifying power, then, of the instrument is the ratio of these two angles; that is,

$$\textit{Magnification} = (ab/f)/(ab/F) = F/f.$$

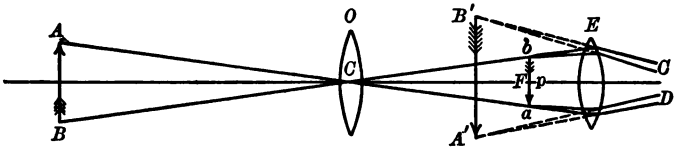


FIG. 164.—Astronomical telescope.

223. Combination of Two Lenses.—Let L and L', Fig. 165, represent two lenses (convex or concave) placed so that their axes coincide. Let d = distance between the lenses; f = focal length of L; and f' = focal length of L'. Consider first the relation of object to image in the case of L. From the general lens equation we may write $1/q - 1/p = 1/f$, where p is the object distance, and q is the image distance, with reference to L. Consider now that the image formed by L becomes the object for L'; then $q \pm d$ = distance of image from $L' = p'$ = object distance with respect to L'. If we let q' be the image distance with respect to L', then $1/q' - 1/p' = 1/f'$. In dealing with these two cases, it is important to note that *distances measured on the side of the lens from which the light comes are positive, and distances measured the other side are negative.*

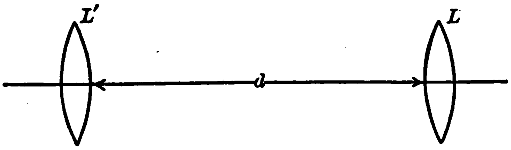


FIG. 165.—Lenses placed coaxially.

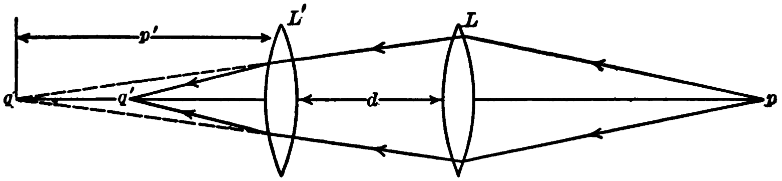


FIG. 166.—Image due to two convex lenses.

When the lenses touch, $d = 0$, and we may write $1/F = 1/f + 1/f'$, where F is the focal length of the combination.

Example.—Two convex lenses of focal lengths $f = 20$, and $f' = 30$ are placed symmetrically on an axis at a distance 10 cm apart, Fig. 166. An object is placed 100 cm in front of lens L. Find the position of the image due to the combination.

Solution.—Let us consider first the image formed by L. Here p is +, f is −, and q is −, hence the equation $1/q - 1/p = 1/f$ *becomes* $1/q + 1/100 = 1/20$*, from which* $q = 25$*. Since q is measured on the negative side of the lens,* $(q + d) = p' = -25 + 10 = -15$*. In the case of lens L′, p′, q′, and f′ are all on the negative side, and hence we write* $-1/q' + 1/15 = -1/30$*, and hence* $q' = 10$ *cm, measured to the left of L′.*

Example.—Given a convex lens ($f = 4$ in.) and a concave lens ($f' = 4$ in.) placed coaxially 4 in. apart, to find the position and character of the image of an object placed 6 in. from lens L, Fig. 167.

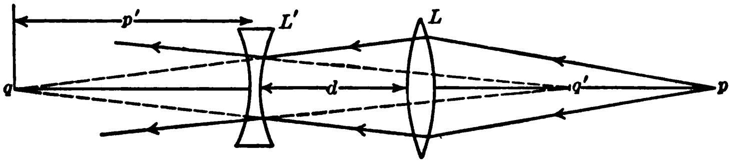


Fig. 167.—Image due to combination of concave and convex lenses.

Solution.—We shall first find the image due to L. In this case p is positive and q and f are negative, hence $1/q + 1/6 = 1/4$*, whence* $q = 12$*, measured on the negative side of L. In the case of the concave lens, q′ and f′ are positive while p′ is negative. The term* $(q + d) = p' = -8$*. The general equation* $1/q - 1/p = 1/f$ *therefore becomes* $1/q' + 1/8 = 1/4$*, from which* $q' = 8$*, and is measured to the right of L′. This means that the image is virtual, and lies between the object and the lens L.*

Problems

The indices of refraction for the B, C, D, F, and H lines of flint glass, crown glass, water and carbon disulphide are given in Table XXX, page 201.

1001. Find for flint glass (*a*) the total dispersion; (*b*) mean dispersion; (*c*) dispersive power.

1002. Find for crown glass (*a*) the total dispersion; (*b*) mean dispersion; (*c*) dispersive power.

1003. Find for water (*a*) the total dispersion; (*b*) mean dispersion; (*c*) dispersive power.

1004. How does the dispersive power of carbon disulphide compare with that of flint glass?

1005. The refracting angle A of flint glass prism is 20°. Find the angular dispersion between (*a*) the B and H lines; (*b*) the C and D lines.

1006. The angular dispersion in a crown glass prism for the B and H lines is 22′ 27.84″. Find the refracting angle.

1007. The refracting angle of CS_2 prism is 40°. Find the length of the spectrum (*B* to *H*) formed on a screen 10 ft. distant from the prism, assuming that the screen is at right angles to the *B* line.

1008. How far from a flint-glass prism having a refracting angle of 30° should a screen be placed so that a spectral image 2 ft. in width shall be shown?

1009. A plano-convex lens whose radius is 60 cm is made of a glass having an index of refraction of 1.53 for red rays, and 1.55 for blue rays. Find how much farther from the lens is the principal focus for red than is that for blue.

1010. Find the angle of a flint glass prism that will achromatize the region from *B* to *H* in a crown-glass prism whose angle is 4°.

1011. The refracting angle of a flint glass prism is 4°. What must be the refracting angle of a crown glass prism that achromatizes the $B - H$ region of the spectrum?

1012. A projecting lantern is to produce a magnification of 50 diameters at a distance of 50 ft. Find the distance of the lens from the slide, and the focal length of the lens.

1013. The projection lens of a lantern has a focal length of 1 ft. How far back of the lens must a slide be placed in order to focus clearly upon the screen 24 ft. from the lantern?

1014. Make drawing of the section of an eye, and a spectacle lens necessary for correction of (*a*) far sight; (*b*) near sight.

1015. If the greatest distance of distinct vision for a myopic (near-sighted) eye is 10 cm, what is the focal length of a spectacle lenses to read at a distance of 25 cm?

1016. If the nearest distance for distinct vision for a far-sighted person is 35 in., what should be the focal length of the spectacles he would require for reading at a distance of 10 in.?

1017. If the greatest distance of distinct vision for a myopic eye is 4 in. what should be the focal length of the proper reading spectacles?

1018. The focal length of the objective of a 12-in. refracting telescope is 20 ft. Determine the focal length of the eyepiece in order that the magnifying power may be 80.

1019. Given two convex lenses of focal lengths 23 in. and 1 in. respectively. Make drawing to illustrate the use of these lenses as a telescope, and compute the magnifying power.

1020. Explain the principle of the compound microscope, with the aid of a diagram, and state from what data you would calculate its magnifying power.

1021. Write a problem to illustrate the magnifying power of a compound microscope, as shown in Fig. 163, and solve the same. State the relative distance of *AB* from *O*, and *ab* from *E*.

1022. Substitute for lens *L*, Fig. 166, a concave lens having the same focal length, and find the position of the image, with respect to *L′*, data as in *Example* under Fig. 166.

1023. A convex and a concave lens, each 10 in. in focal length, are held coaxially at a distance of 3 in. apart. Find the position of the image when the object is at a distance of 15 in. beyond the convex lens.

1024. Solve problem 1023 assuming the object to be placed 15 in. beyond the concave lens.

1025. Two thin convex lenses, having a common axis, touch. The focal length of one is 20 cm; that of the other is 15 cm. Find the focal length of the combination.

APPENDIX

TABLES

I. SOME IMPORTANT EQUIVALENTS AND EQUATIONS

NOTE.—The approximate values employed in ordinary computations are given in parentheses.

$\pi = 3.1416$; $\pi^2 = 9.8696 = (9.87)$; $\sqrt{\pi} = 1.7724$; $\log \pi = 0.49715$.

Conversion factor between common and natural logs = 2.30258 = (2.3).

$\mathrm{Log}_{10} n = \log_e n / 2.3$; $\log_e n = \log_{10} n \times 2.3$.

One radian = 57°.296 = (57°.3); 1° = 0.01745 radian.

$\sin 0° = 0.0$
$\sin 30° = 0.5$
$\sin 45° = 0.707$
$\sin 60° = 0.866$
$\sin 90° = 1.0$

$\cos 0° = 1.0$
$\cos 30° = 0.866$
$\cos 45° = 0.707$
$\cos 60° = 0.5$
$\cos 90° = 0.0$

Circle: *circumference* $= 2\pi r$; *area* $= \pi r^2$.

Cylinder: *lateral area* $= 2\pi rh$; *volume* $= \pi r^2 h$.

Cone: *lateral area* = ½(circumference × slant height); *volume* = ⅓ (area base × height).

Sphere: *area* $= 4\pi r^2$; *volume* $= \frac{4}{3}\pi r^3$.

Pressure of one atmosphere = 76 cm mercury = 30 in. mercury = 1,033.3 g per cm² = 1,012,634 dynes per cm² = 14.7 lb. per in².

Mass of 1 cm³ of pure water at 4°C = 1 gram (very nearly).

Mass of 1 cu. ft. of pure water at 4°C = 62.3565 lb. = (62.4 lb.).

Mass of 1 gallon of pure water at 4°C = 8.33585 lb. = (8.3 lb.).

II. CONVERSION TABLES

English to Metric			Metric to English		
1 mile	=	1.60935 km	1 kilometer	=	0.62137 mi.
1 mile	=	1,609.347 m	1 meter	=	0.0006214 mi.
1 foot	=	0.3048 m	1 meter	=	3.28083 ft.
1 inch	=	2.54 cm	1 centimeter	=	0.3937 in.
1 cubic foot	=	28.31701 l	1 liter	=	0.03532 cu. ft.
1 qt. (dry)	=	1.101 l	1 liter	=	0.908 qt. (dry)
1 qt. (liq'd)	=	0.946 l	1 liter	=	1.0567 qt. (liquid)
1 pound	=	0.45359 kg	1 kilogram	=	2.204622 lb.
1 grain	=	0.06480 g	1 gram	=	15.432 gr.

III. VALUES OF g

Boston, Mass.	980.38	Washington, D. C.	980.10
Ithaca, N. Y.	980.29	Cincinnati, O.	979.99
Chicago, Ill.	980.26	Charlottesville, Va.	979.92
Cleveland, O.	980.23	Denver, Col.	979.60
Philadelphia, Pa.	980.18	Pike's Peak, Col.	978.94

IV. MOMENTS OF INERTIA

1. Uniform thin rod, axis through middle, $I = Ml^2/12$.
2. Rectangular figure, axis through center, perpendicular to plane, width a, length b, $I = M(a^2 + b^2)/12$.
3. Circular plate or cylinder, axis through center, perpendicular to face, radius r, $I = Mr^2/2$.
4. Circular ring, axis through center, perpendicular to face, outer radius $= r$, inner radius $= r'$, . . . $I = M(r^2 + r'^2)/2$.
5. Sphere, axis through center, radius r, $I = Mr^2 2/5$.
6. Moment of inertia about axis parallel to axis through center of gravity, $I = I_0 + Md^2$.
7. Moment of force $Fd = I\alpha = Ia/r$.

V. ELASTIC CONSTANTS

	Young's Modulus		Simple Rigidity		Volume Elasticity
	Dynes per cm²	Lb. per in. sq.	Dynes per cm²	Lb. per in. sq.	Dynes per cm²
Aluminum	7×10^{11}	9.5×10^6	3×10^{11}	4×10^6	7.5×10^{11}
Brass	10×10^{11}	14×10^6	4×10^{11}	5.4×10^6	10.5×10^{11}
Copper	12×10^{11}	17×10^6	4.5×10^{11}	6.5×10^6	12.5×10^{11}
Cast iron	13×10^{11}	18×10^6	5.6×10^{11}	7.8×10^6	9.5×10^{11}
Wrought iron	20×10^{11}	28×10^6	7.7×10^{11}	11×10^6	16.5×10^{11}
Steel	22×10^{11}	31×10^6	8×10^{11}	12×10^6	18.5×10^{11}
Water	....	....	....	....	0.22×10^{11}

VI. DENSITIES

Air at 0°C and 6 cm	0.00129
Alcohol	0.8
Aluminum	2.6
Antimony	6.7
Beeswax	0.9
Bismuth	9.8
Brass	8.5
Coal	1.3 to 1.8
Copper	8.9
Cotton-seed oil	0.926
Diamond	3.5
Ether	0.74
German silver	8.4
Glass, crown	2.6
Glass, flint	3.7
Glycerine	1.26
Gold	19.3
Granite	2.7
Human body	0.9
Hydrochloric acid	1.27
Ice	0.9
Illuminating gas	0.001
Iron, wrought	7.8
Ivory	1.8
Lead	11.3
Magnesium	1.7
Marble	2.7
Mercury, at 0°C	13.596
Milk	1.03
Nickel	8.9
Nitric acid	1.56
Olive oil	0.92
Paraffin	0.9
Platinum	21.5
Silver	10.5
Steel	7.
Sulphuric acid	1.84
Sulphur	2.0
Sugar	1.6
Tin	7.3
Turpentine	0.873
Water, at 0°C	0.999
Water, at 4°C	1.00
Water, sea	1.03
Wood, beech	0.7 to 0.9
Wood, maple	0.6 to 0.8
Wood, pine	0.4 to 0.5
Wood, lignum vitæ	1.3
Zinc	7.2

VII. SURFACE TENSIONS

Liquid	T in degrees per cm in contact with		
	Air	Water	Mercury
Water	75	0	430
Mercury	535	418	0
Olive oil	37	21	347
Turpentine	29	12	241
Alcohol	26	....	400

VIII. DIFFUSION CONSTANTS

NOTE.—The concentration n is expressed in gram-molecules per liter; t is the temperature; and k is the diffusion constant, expressed in grams per cm², per day.

Substance	n	$t°$	k
Hydrochloric acid................	1.0	5°	1.74
Hydrochloric acid................	1.0	12°	2.09
Sodium chloride..................	1.0	5°	0.76
Sodium chloride..................	1.0	10°	0.91
Sugar............................	1.0	9°	0.31
Albumen..........................	1.0	13°	0.06
Urea.............................	1.0	12°	0.81

IX. COEFFICIENTS OF LINEAR EXPANSION

Aluminum........	0.000023	Iron and steel.....	0.000012
Brass.............	0.000018	Lead.............	0.000028
Copper...........	0.000017	Platinum.........	0.000008
Glass.............	0.000008	Silver............	0.000019
Gold.............	0.000014	Zinc..............	0.000029

X. COEFFICIENTS OF VOLUME EXPANSION, SOLIDS

NOTE.—The coefficients of volume expansion of the solids named in Table IX may be found by multiplying the linear coefficients by three; that is $\beta = 3\alpha$.

XI. COEFFICIENTS OF VOLUME EXPANSION, LIQUIDS

NOTE.—$V_t = V_0(1 + \beta t + \beta' t^2 + B'' t^3)$.

Substance	β	β'	β''
Alcohol, ethyl....................	$1{,}020 \times 10^{-6}$	220×10^{-8}	....
Alcohol, methyl..................	$1{,}134 \times 10^{-6}$	136×10^{-8}	87×10^{-10}
Benzol...........................	$1{,}176 \times 10^{-6}$	128×10^{-8}	80×10^{-10}
Ether............................	$1{,}500 \times 10^{-6}$	350×10^{-8}	40×10^{-10}
Pentane..........................	$1{,}465 \times 10^{-6}$	310×10^{-8}	16×10^{-10}

XII. SPECIFIC HEATS

Air, constant pressure	0.237	Iron	0.116
Alcohol	0.602	Lead	0.03
Aluminum	0.22	Marble	0.21
Brass	0.094	Mercury	0.033
Copper	0.094	Silver	0.056
Glycerine	0.55	Steam, 100°C, 76 cm	0.48
Glass	0.2	Water	1.0
Ice	0.5	Zinc	0.094

XIII. SPECIFIC HEAT SUPERHEATED STEAM

Pressure in lb. per sq. in.	14.2	50	100	200
Temp. 300°F	0.46	0.51	.....	.....
Temp. 400°F	0.46	0.50	0.56	0.68
Temp. 500°F	0.46	0.49	0.53	0.59
Temp. 600°F	0.46	0.49	0.51	0.55

XIV. MELTING POINTS

Mercury	−38.8°	Aluminum	657°
Phosphorus	44.3	Silver	961
Sulphur	115	Gold	1,063
Tin	232	Copper	1,084
Bismuth	260	Iron	1,100
Cadmium	320	Platinum	1,778
Lead	327	Iridium	2,200
Zinc	419	Tungsten	2,950

XV. BOILING POINTS

Ethylene	− 103°	Alcohol	78°
Ammonia	−38.5	Benzene	80
Chlorine	−33.6	Toluene	110
Ether	35	Turpentine	160
Carbon bisulphid	46	Glycerine	290
Chloroform	61	Mercury	357

XVI. BOILING POINTS OF WATER UNDER DIFFERENT PRESSURES

73 cm	98.88°	76 cm	100.00°
74 cm	99.26	77 cm	100.37
75 cm	99.63	78 cm	100.73

XVII. EXTREMELY LOW FREEZING AND BOILING POINTS

	Freezing Point	Boiling Point
Nitrogen	−210°C	−194.0°C
Oxygen	−227	−181.0
Hydrogen	−260	−252.5
Helium		−268.8

XVIII. NUMBER OF GRAMS OF WATER VAPOR REQUIRED TO SATURATE THE AIR, PER CUBIC METER

− 10°C	2.363 g	5°C	6.761 g	20°C	17.118 g
− 9	2.546	6	7.219	21	18.143
− 8	2.741	7	7.703	22	19.222
− 7	2.949	8	8.215	23	20.355
− 6	3.171	9	8.757	24	21.546
− 5	3.407	10	9.330	25	22.796
− 4	3.659	11	9.935	26	24.109
− 3	3.926	12	10.574	27	25.487
− 2	4.211	13	11.249	28	26.933
1	4.513	14	11.961	29	28.450
0	4.835	15	12.712	30	30.039
1	5.176	16	13.505	31	31.704
2	5.538	17	14.339	32	33.449
3	5.922	18	15.218	33	35.275
4	6.330	19	16.144	34	37.187

XIX. HEATS OF COMBUSTION IN CALORIES PER GRAM

Hydrogen	34,700	Alcohol	7,183
Gunpowder	700	Illuminating gas	6,000
Dynamite	1,300	Wood	about 4,300
Sulphur	2,200	Anthracite coal	8,000

XX. HEATS OF COMBUSTION IN B.T.U. PER POUND

Bituminous Coal		Semi-bituminous Coal	
Streator, Ill.	13,700	Blassburg, Pa.	13,500
Wilmington, Ill.	14,000	Pocahontas, W. Va.	15,700
Saginaw, Mich.	13,500	Cumberland, Md.	16,300
Hocking Valley, O.	14,000		
Jackson, O.	14,000	Anthracite Coal	
Turtle Creek, Pa.	15,000	Lackawanna	13,900
Youghiogheny, Pa.	15,000	Lykens Valley	13,700
Thacker, W. Va.	15,200	Scranton	13,800

Carbon burned to CO_2	14,650	Sulphur burned to SO_2	4,000
Carbon burned to CO	4,400	Marsh gas to CH_4	23,500
Hydrogen burned to H_2O	62,100	Wood, hard	8,600

XXI. THERMAL CONDUCTIVITY

NOTE.—In this table k is the number of calories which will pass per second through 1 cm^2 of a plate 1 cm thick, the difference of temperature on the two sides being 1°C.

	k		k
Air	0.00005	Ice	0.005
Aluminum	0.5	Mercury	0.016
Copper	0.9	Platinum	0.17
Cotton	0.00004	Pine wood	0.0002
Felt	0.00009	Silver	1.0
Glass	0.0015	Snow	0.0001
Gold	0.7	Water	0.0015
Iron	0.15	Zinc	0.25

XXII. THERMAL CONDUCTIVITY

NOTE.—In this table k is the number of B.t.u.'s which will pass per hour through 1 sq. ft. of a plate 1 in. thick, the difference of temperature on the two sides being 1°F.

	k		k
Copper	515.0	Glass	7.0
Iron	233.0	Brickwork	5.0
Lead	113.0	Plaster	4.0
Stone	17.0	Pine wood	0.75

XXIII. ATOMIC WEIGHTS

Element	Atomic Weight	Valence	Element	Atomic Weight	Valence
Aluminum	27.0	3	Lithium	7.0	1
Antimony	120.0	3, 5	Magnesium	24.3	2
Arsenic	75.0	3, 5	Manganese	55.0	2, 3, 7
Barium	137.4	2	Mercury	200.0	1, 2
Bismuth	208.0	3, 5	Nickel	58.7	2, 3
Boron	11.0	3	Nitrogen	14.0	3, 5
Bromine	80.0	1	Oxygen	16.0	2
Cadmium	112.4	2	Palladium	106.7	2, 4
Calcium	40.0	2	Phosphorus	31.0	3, 5
Carbon	12.0	4	Platinum	195.2	2, 4
Chlorine	35.45	1	Potassium	39.0	1
Chromium	52.0	2, 3, 6	Radium	226.4	2
Cobalt	59.0	2, 3	Selenium	79.0	2, 4, 6
Copper	63.57	1, 2	Silicon	28.0	4
Fluorine	19.0	1	Silver	107.88	1
Gold	197.2	1, 3	Sodium	23.0	1
Helium	4.0	.....	Strontium	87.6	2
Hydrogen	1.0	1	Sulphur	32.0	2, 4, 6
Iodine	126.9	1	Tin	119.0	2, 4
Iridium	193.0	4	Tungsten	184.0	6
Iron	55.8	2, 3	Uranium	238.5	4, 6
Lead	207.0	2, 4	Zinc	65.37	2

XXIV. RESISTIVITY VALUES AT 0°C

Conductor	Ohm-cm	Ohms per mil-foot	Conductor	Ohm-cm	Ohms per mil-foot
Aluminum.....	2.6×10^{-6}	17.5	Silver..........	1.5×10^{-6}	9.5
Copper........	1.6×10^{-6}	9.5	Manganin 20°C	42×10^{-6}	260.0
Gold...........	2.2×10^{-6}	12.6	Mercury.......	94×10^{-6}	566.0
Iron...........	9.7×10^{-6}	58.3	Nickel	12×10^{-6}	75.0
German silver..	20.9×10^{-6}	125.7	Platinum......	9×10^{-6}	54.0

XXV. WIRE GAGE VALUES, AMERICAN (B. & S.)

Gage No.	Diam. in mm	Diam. in mils	Sq. of Diam. mils	Gage No.	Diam. in mm	Diam. in mils	Sq. of Diam. in mils
0000	11.684	460.00	211,600.0	19	0.899	35.39	1252.4
000	10.405	409.64	167,805.0	20	0.812	31.96	1021.5
00	9.266	364.80	133,079.4	21	0.723	28.46	810.1
0	8.254	324.95	105,592.5	22	0.644	25.35	642.7
1	7.348	289.30	83,694.2	23	0.573	22.57	509.5
2	6.544	257.63	66,373.0	24	0.511	20.10	404.0
3	5.827	229.42	52,634.0	25	0.455	17.90	320.4
4	5.189	204.31	41,742.0	26	0.405	15.94	254.0
5	4.621	181.94	33,102.0	27	0.361	14.19	201.5
6	4.115	162.02	26,250.5	28	0.321	12.64	159.8
7	3.665	144.28	20,816.0	29	0.286	11.26	126.7
8	3.264	128.49	16,509.0	30	0.255	10.03	100.5
9	2.907	114.43	13,094.0	31	0.227	8.93	79.7
10	2.588	101.89	10.381.0	32	0.202	7.95	63.2
11	2.305	90.74	8,234.0	33	0.180	7.08	50.1
12	2.053	80.81	6,529.9	34	0.160	6.30	39.7
13	1.828	71.96	5,178.4	35	0.143	5.61	31.5
14	1.628	64.01	4,106.8	36	0.127	5.00	25.0
15	1.450	57.07	3,256.7	37	0.113	4.45	19.8
16	1.291	50.82	2,582.9	38	0.101	3.96	15.7
17	1.150	45.26	2,048.2	39	0.090	3.53	12.5
18	1.024	40.30	1,624.3	40	0.080	3.14	9.9

XXVI. THERMOELECTRIC POWERS IN MICROVOLTS PER DEGREE

Bismuth................	−89	Zinc..................	+3.7
Cobalt..................	−22	Copper	+3.8
German silver...........	−12	Iron..................	+17.5
Platinum	−1	Constantan	+19.3
Lead....................	0	Antimony.............	+24
Silver..................	+3	Selenium	+807

XXVII. MAGNETIC PERMEABILITY

	Soft Iron		Cast Iron		Steel	
H	*B*	μ	*B*	μ	*B*	μ
5	10,050	2,010	3,000	600	750	150
10	12,550	1,255	5,000	500	1,650	165
20	14,550	727	6,000	300	5,875	294
30	15,200	507	6,500	217	9,875	329
40	15,800	395	7,100	177	11,600	290
50	16,000	320	7,350	149	12,000	240

XXVIII. DIELECTRIC CONSTANTS, *k*

Vacuum	1.0000	Mica	6
Air	1.0006	Paper	2
Glass, crown	7.0	Paraffin	2
Glass, flint	8.0	Sulphur	3
Gutta percha	4.0	Water	81

XXIX. INDICES OF REFRACTION

Water	1.33	Benzene	1.50
Carbon bisulphide	1.64	Crown glass	1.52
Turpentine	1.47	Flint glass	1.62
Alcohol	1.36	Diamond	2.47

XXX. INDICES OF REFRACTION FOR CERTAIN LINES

	B	*C*	*D*	*F*	*H*
Flint glass	1.6127	1.6144	1.6193	1.6315	1.6527
Crown glass	1.5301	1.5311	1.5339	1.5404	1.5509
Water (18°.7 C)	1.3310	1.3320	1.3336	1.3380	1.3448
CS_2 (18°.7 C)	1.6182	1.6219	1.6308	1.6555	1.7020

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